Worksheet # 21: Optimization

1. Suppose that f is a function on an open interval $I = (a, b)$ and c is in I. Suppose that f is continuous at c, $f'(x) > 0$ for $x > c$ and $f'(x) < c$ for $x < c$. Is $f(c)$ an absolute minimum value for f on I? Justify your answer.

2. Find the point(s) on the hyperbola $y = \frac{16}{x}$ that is (are) closest to $(0, 0)$. Be sure to clearly state what function you choose to minimize or maximize and why.

3. A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

4. A hockey team plays in an arena with a seating capacity of 15000 spectators. With the ticket price set at \$12, average attendance at a game has been 11000. A market survey indicates that for each dollar the ticket price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price to maximize their revenue from ticket sales?

5. An oil company needs to run a pipeline to a nearby station. The station and oil company are on opposite sides of a river that is 1 km wide, and that runs exactly west-east. Also, the station is 10 km east of where the oil company would be if it was on the same side of the river. The cost of land pipe is \$200 per meter and the cost of water pipe is \$300 per meter. Set up an equation whose solution(s) are the critical points of the cost function for this problem.

6. A 10 ft length of rope is to be cut into two pieces to form a square and a circle. How should the rope be cut to maximize the enclosed area?

7. Consider a can in the shape of a right circular cylinder. The top and bottom of the can is made of a material that costs 4 cents per square centimeter, and the side is made of a material that costs 3 cents per square centimeter. We want to find the dimensions of the can which has volume 72 π cubic centimeters, and whose cost is as small as possible.

   (a) Find a function $f(r)$ which gives the cost of the can in terms of radius r. Be sure to specify the domain.

   (b) Give the radius and height of the can with least cost.

   (c) Explain how you known you have found the can of least cost.

8. A box is to have a square base, no top, and a volume of 10 cubic centimeters. What are the dimensions of the box with the smallest possible total surface area? Provide an exact answer; do not convert your answer to decimal form. Make a sketch and introduce all the notation you are using.