Worksheet # 23: Approximating Area

1. Write each of following in summation notation:

   (a) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

   (b) $2 + 4 + 6 + 8 + 10 + 12 + 14$

   (c) $2 + 4 + 8 + 16 + 32 + 64 + 128$.

2. Compute $\sum_{i=1}^{4} \left(\sum_{j=1}^{3} (i + j) \right)$.

   The following summation formulas will be useful below.

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}, \qquad \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Find the number n such that $\sum_{i=1}^{n} i = 78$.

4. Give the value of the following sums.

   (a) $\sum_{j=1}^{20} (2k^2 + 3)$

   (b) $\sum_{j=11}^{20} (3k + 2)$

5. The velocity of a train at several times is shown in the table below. Assume that the velocity changes linearly between each time given.

t=time in minutes	0	3	6	9
v(t)=velocity in Km/h	20	80	100	140

   (a) Plot the velocity of the train versus time.

   (b) Compute the left and right-endpoint approximations to the area under the graph of v.

   (c) Explain why these approximate areas are also an approximation to the distance that the train travels.

6. Let $f(x) = 1/x$. Divide the interval $[1, 3]$ into five subintervals of equal length and compute R_5 and L_5, the left and right endpoint approximations to the area under the graph of f in the interval $[1, 3]$. Is R_5 larger or smaller than the true area? Is L_5 larger or smaller than the true area?

7. Let $f(x) = \sqrt{1 - x^2}$. Divide the interval $[0, 1]$ into four equal subintervals and compute L_4 and R_4, the left and right-endpoint approximations to the area under the graph of f. Is R_4 larger or smaller than the true area? Is L_4 larger or smaller than the true area? What can you conclude about the value π?

8. Let $f(x) = x^2$.

   (a) If we divide the interval $[0, 2]$ into n equal intervals of equal length, how long is each interval?

   (b) Write a sum which gives the right-endpoint approximation R_n to the the area under the graph of f on $[0, 2]$.

   (c) Use one of the formulae for the sums of powers of k to find a closed form expression for R_n.

   (d) Take the limit of R_n as n tends to infinity to find an exact value for the area.