Worksheet # 24: Definite Integrals and The Fundamental Theorem of Calculus

1. Suppose $\int_0^1 f(x)\,dx = 2$, $\int_1^2 f(x)\,dx = 3$, $\int_0^1 g(x)\,dx = -1$, and $\int_0^2 g(x)\,dx = 4$.

   Compute the following using the properties of definite integrals:

   (a) $\int_1^2 g(x)\,dx$

   (b) $\int_0^2 [2f(x) - 3g(x)]\,dx$

   (c) $\int_1^1 g(x)\,dx$

   (d) $\int_1^2 f(x)\,dx + \int_2^0 g(x)\,dx$

   (e) $\int_0^2 f(x)\,dx + \int_2^1 g(x)\,dx$

2. Simplify $\int_0^2 3f(x)\,dx + \int_1^3 3f(x)\,dx - \int_0^3 2f(x)\,dx - \int_1^2 3f(x)\,dx$

3. Find $\int_0^5 f(x)\,dx$ where $f(x) = \begin{cases} 3 & \text{if } x < 3 \\ x & \text{if } x \geq 3 \end{cases}$.

4. (a) State both parts of the Fundamental Theorem of Calculus using complete sentences.

   (b) Consider the function $f(x)$ on $[1,\infty)$ defined by $f(x) = \int_1^x \sqrt{t^5 - 1}\,dt$. Argue that f is increasing.

   (c) Find the derivative of the function $g(x) = \int_1^{x^3} \sqrt{t^5 - 1}\,dt$ on $(1, \infty)$.

5. Use Part I of the Fundamental Theorem of Calculus to find the derivative of the following functions:

   (a) $g(x) = \int_1^x \left(2 + t^4\right)^5\,dt$

   (b) $F(x) = \int_x^4 \cos\left(t^5\right)\,dt$

   (c) $h(x) = \int_0^{x^2} \sqrt[3]{1 + r^3}\,dr$

   (d) $y(x) = \int_{\frac{1}{x^2}}^0 \sin^3(t)\,dt$

   (e) $G(x) = \int_{\sqrt{x}}^{x^2} \sqrt{t}\sin(t)\,dt$

6. Use Part II of the Fundamental Theorem of Calculus to evaluate the following integrals or explain why the theorem does not apply:

   (a) $\int_{-2}^5 6x\,dx$

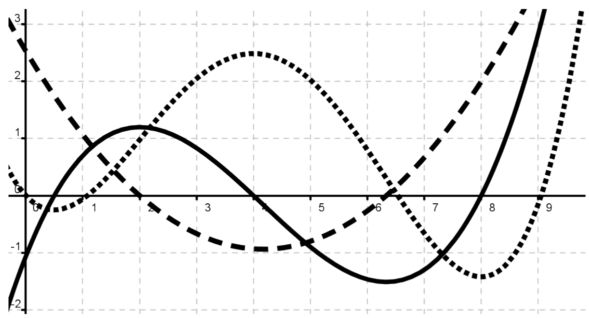
(b) $\displaystyle\int_{-2}^{7} \frac{1}{x^5}\,dx$

(c) $\displaystyle\int_{-1}^{1} e^{u+1}\,du$

(d) $\displaystyle\int_{0}^{\frac{\pi}{4}} \sec^2(t)\,dt$

(e) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sin(2x)}{\sin(x)}\,dx$

7. Below is pictured the graph of the function $f(x)$, its derivative $f'(x)$, and an antiderivative $\int f(x)\,dx$. Identify $f(x), f'(x)$ and $\int f(x)\,dx$.



8. Evaluate the following limits by first recognizing the sum as a Riemann sum:

(a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$

(b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sqrt{3+\frac{i}{n}}}{n}$

(c) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} 2\frac{(2+\frac{2i}{n})^2}{n}$