Worksheet # 29: Review II for Final

1. Compute the following limits.

   (a) $\lim_{t\to 9} \dfrac{9-t}{3-\sqrt{t}}$

   (b) $\lim_{\theta\to 0} \dfrac{\sin(3\theta)}{10\theta}$

   (c) $\lim_{x\to\infty} \dfrac{e^x}{x^2}$

   (d) $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x}$

2. (a) State the limit definition of the continuity of a function f at $x = a$.

   (b) State the limit definition of the derivative of a function f at $x = a$.

   (c) Given $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 4-3x & \text{if } x \geq 1 \end{cases}$. Is the function continuous at $x = 1$? Is the function differentiable at $x = 1$? Use the definition of the derivative. Graph the function to check your answer.

3. Provide the most general antiderivative of the following functions:

   (a) $f(x) = x^4 + x^2 + x + 1000$

   (b) $g(x) = (3x-2)^{20}$

   (c) $h(x) = \dfrac{\sin(\ln(x))}{x}$

4. Use implicit differentiation to find $\dfrac{dy}{dx}$, and compute the slope of the tangent line at (1,2) for the following curves:

   (a) $x^2 + xy + y^2 + 9x = 16$

   (b) $x^2 + 2xy - y^2 + x = 2$

5. An rock is thrown up the in the air and returns to the ground 4 seconds later. What is the initial velocity? What is the maximum height of the rock? Assume that the rock's motion is determined by the acceleration of gravity, 9.8 meters/second^2.

6. A conical tank with radius 5 meters and height 10 meters is being filled with water at a rate of 3 cubic meters per minute. How fast is the water level increasing when the water's depth is 3 meters?

7. A rectangular storage container with an open top is to have a volume of 10 cubic meters. The length of the container is twice its width. Material for the base costs \$10 per square meter while material for the sides costs \$6 per square meter. Find the cost of materials for the least expensive possible container.

8. (a) State the Mean Value Theorem.

   (b) If $3 \leq f'(x) \leq 5$ for all x, find the maximum possible value for $f(8) - f(2)$.

9. Use linearization to approximate $\cos(\frac{11\pi}{60})$

   (a) Write down $L(x)$ at an appropriate point $x = a$ for a suitable function $f(x)$.

   (b) Use part(a) to find an approximation for $\cos(\frac{11\pi}{60})$

   (c) Find the absolute error in your approximation.

10. Find the value(s) c such that $f(x)$ is continuous everywhere.

$$f(x) = \begin{cases} (cx)^3 & \text{if } x < 2 \\ \ln(x^c) & \text{if } x \geq 2 \end{cases}$$

11. (a) Find y' if $x^3 + y^3 = 6xy$.
    (b) Find the equation of the tangent line at $(3, 3)$.

12. Show that the function $f(x) = 3x^5 - 20x^3 + 60x$ has no absolute maximum or minimum.

13. Compute the following definite integrals:

    (a) $\int_{-1}^{1} e^{u+1}\, du$

    (b) $\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

    (c) $\int_{1}^{9} \frac{x-1}{\sqrt{x}}\, dx$

    (d) $\int_{0}^{10} |x - 5|\, dx$

    Hint: For some of the integrals, you will need to interpret the integral as an area and use facts from geometry to compute the integral.

14. Write as a single integral in the form $\int_a^b f(x)\, dx$:

$$\int_{-2}^{2} f(x)\, dx + \int_{2}^{5} f(x)\, dx - \int_{-2}^{-1} f(x)\, dx$$