IN-CLASS GROUP WORK
FRIDAY, JAN 28, 2011 - MA 614

1. Catalan numbers

Prove that the number of lattice paths from $(0,0)$ to (n,n) in $\mathbb{Z}^2$ with steps $(1,0)$ and $(0,1)$ whose edges lie below the line $x = y$ is $\frac{1}{n+1}\binom{2n}{n}$. Such paths are called *Dyck paths of length* $2n$.

2. A bijection

Prove that the set of length $2n$ Dyck paths is in bijection with the set of *Dyck words of length* $2n$, i.e. strings of n X's and n Y's such that no initial segment of the string has more Y's than X's.

3. A recursion

Prove that the Catalan numbers satisfy the recursion

$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$$

for $n \geq 0$.