MA 614 – Homework 12
Due Friday, February 11

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. In this exercise we first use local data about a set of combinatorial objects to define an ogf, and then use another ogf to study the first one. Consider the set A_n of lattice paths from $(0,0)$ to $(n,0)$ that do not cross below the x-axis and use only the steps $(1,1)$, $(1,-1)$, and $(1,0)$. Define the weight of a path P to be $w_P(t) = t^k$ where k is the number of $(1,0)$ steps in P and t is a variable. Let $a_n(t)$ be $\sum_{P \in A_n} w_P(t)$.

   (a) What is $a_n(t)$ the ogf for?

   (b) Find the ogf $\sum_{n \geq 0} a_n(2)x^n$.

   (c) Using this ogf, prove that $a_{n-1}(2) = \frac{1}{n+1}\binom{2n}{n}$, the n-th Catalan number.

2. Let $f(n)$ be the number of tilings of a $3 \times 2n$ rectangle by dominos (where a domino is a 1×2 or 2×1 rectangle). Find the ogf $\sum_{n \geq 0} f(n)x^n$.