

**MA 614 – Homework 16**  
**Due Monday, Feb 21**

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. For  $n \geq 1$ , prove that

$$\prod_{k=0}^{n-1} (1 + q^k x) = \sum_{k=0}^n q^{\binom{k}{2}} \begin{bmatrix} n \\ k \end{bmatrix}_q x^k .$$

NOTE: This shows that one useful  $q$ -analogue of  $f_n(x) = (1 + x)^n$  is

$$f_n(x, q) = \prod_{k=0}^{n-1} (1 + q^k x) .$$

2. Let  $x$  and  $y$  be variables satisfying the commutation relation  $qxy = yx$ , where  $q$  commutes with  $x$  and  $y$ . Show that

$$(x + y)^n = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q x^k y^{n-k} .$$