MA 614 – Homework 18
Due Friday, Feb 25

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Let $S(n,k)$ denote the Stirling numbers of the second kind. For $n \geq 1$, prove directly the following (mentioned at the end of page 81 and beginning of page 82 in EC1):

   (a) $S(n,2) = 2^{n-1} - 1$

   (b) $S(n,n-1) = \binom{n}{2}$

   (c) $S(n,n-2) = \binom{n}{3} + 3\binom{n}{4}$

2. On page 82 of EC1, there are six identities related to Stirling and Bell numbers, labeled (1.94a)-(1.94f). The proofs given in the text are very succinct. Provide clear proofs for each of these six identities, following the arguments given by Stanley.

3. For n, a, b non-negative integers, the *Chu-Vandermonde* identity is

$$\sum_{i=0}^{n} \binom{a}{i}\binom{b}{n-i} = \binom{a+b}{n}.$$

   State and prove a q-analogue of this identity.

   SUGGESTION: Take a gander at Example 1.1.17 in EC1. Feel inspired by Homework 16.