

**MA 614 – Homework 19**  
**Due Monday, Feb 28**

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Prove the partition identity

$$\prod_{i \geq 1} \frac{1}{(1 - xq^i)} = \sum_{k \geq 0} \frac{x^k q^k}{(1 - q)(1 - q^2) \cdots (1 - q^k)}.$$

NOTE and SUGGESTION: This is a multi-variable relative of the identity (1.77) on page 68 of EC1. From this, you know what the  $q$ 's are recording. Contemplate what the  $x$ 's are recording.

2. Prove the partition identity

$$\prod_{i \geq 1} (1 + qx^{2i-1}) = \sum_{k \geq 0} \frac{x^{k^2} q^k}{(1 - x^2)(1 - x^4) \cdots (1 - x^{2k})}.$$

NOTE and SUGGESTION: This is a multi-variable relative of the identity (1.80) on page 69 of EC1. Contemplate Figure 1.16.