MA 614 – Homework 23
Due Friday, March 25

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Assume $m \leq r \leq n$. Use an inclusion-exclusion argument to prove that

$$\sum_{k=0}^{m} (-1)^k \binom{m}{k} \binom{n-k}{r} = \binom{n-m}{r-m}.$$

2. Prove that $\binom{n}{k}$, $k \geq 0$ and n fixed, is log-concave using simple algebraic calculations.

3. Fix $k \geq 1$. How many permutations of $[n]$ have no cycle of length k? If $f_k(n)$ denotes this number, find $\lim_{n\to\infty} \dfrac{f_k(n)}{n!}$.