MA 614 – Homework 24
Due Tuesday, March 29, by 5PM
SEND VIA EMAIL

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. The *symmetric difference* of two finite sets A and B is $A\Delta B = (A \cup B) - (A \cap B)$. Let X be the set of subsets of $[n]$, signed by $w(T) = (-1)^{|T|}$ for $T \in X$.

   (a) Prove that $\pi(T) = T\Delta\{n\}$ is a sign-reversing involution on X signed by w.

   (b) Using this sign-reversing involution, prove that $\sum_{k\geq 0}(-1)^k\binom{n}{k} = 0$.

2. Prove the following equality using a sign-reversing involution:

$$\sum_{i=k}^{n} S(n,i)s(i,k) = 0$$

   for $k < n$ where the terms involved are Stirling numbers.

3. Use a sign-reversing involution to prove that $\begin{bmatrix} 2n \\ 2k \end{bmatrix}_q$ evaluated at $q = -1$ is equal to $\binom{n}{k}$.