MA 614 – Homework 8
Due Wednesday, February 2

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Use Taylor's formula (from Calc II) to show the power series expansion

$$(1+x)^{\lambda} = \sum_{n \geq 0} \frac{\lambda \cdot (\lambda-1) \cdot (\lambda-2) \cdots (\lambda-n+1)}{n!} x^n$$

   where λ is any complex number. Note that this is the Generalized Binomial Theorem. It is by composing this series with $F(x)$ that one can motivate Example 1.1.10 from EC1.

2. Let $f(x)$ be the ogf, i.e. the ordinary generating function, for the sequence $(a_n)_{n \geq 0}$. Express, using $f(x)$, the ogf for the sequence $a_0, a_0 + a_1, a_0 + a_1 + a_2, \ldots$, i.e. $\left(\sum_{i=0}^{k} a_i\right)_{k \geq 0}$.

3. Find the ogf's for the following sequences.

   (a) $(a_n)_{n \geq 0}$ where a_n is the number of compositions using parts from $\{1, 3\}$.

   (b) $(b_n)_{n \geq 0}$ where b_n is the number of compositions using parts from $\{1, 2, 3\}$.

   (c) $(d_n)_{n \geq 0}$ where d_n is the number of compositions using parts from $\{1, 2, 3\}$ where the 2's are given one of five colors.