MA 614 – Homework 9
Due Friday, February 4

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. For $n \geq 0$, let a_n be the number of compositions of n using positive integers where every part is odd. (We choose by convention that $a_0 = 1$, since there is 1 empty composition).
   (a) Find the ogf for $(a_n)_{n\geq 0}$.
   (b) Using the resulting ogf, prove that $a_n = F_{n-1}$, the $(n-1)$-st Fibonacci number.
   (c) Where have you seen this result before?
2. For $n \geq 0$, let a_n be the number of compositions of n using positive integers where every part is greater than or equal to two. (We again choose by convention that $a_0 = 1$, since there is 1 empty composition).
   (a) Find the ogf for $(a_n)_{n\geq 0}$.
   (b) Using the resulting ogf, express a_n in terms of Fibonacci numbers.
3. Consider the sequence defined by $a_n = \sum_c c_1 \cdot c_2 \cdots c_k$ where $c = (c_1, c_2, \ldots, c_k)$ ranges over all compositions of n. Find the generating function $\sum_{n\geq 0} a_n x^n$.