MA 614 – Enumerative Combinatorics
Spring 2011

1. General Information

Dr. Benjamin Braun
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Office Phone: 257-6810
Class Time/Location: 12:00-12:50PM, MWF, CB 347
Office Location/Hours: 831 POT, 10AM Mon and Fri, 9AM Wed, or by appointment

Note: I reserve the right to change or amend this syllabus at any time for any reason.

2. Course Description

> Having vegetated on the fringes of mathematical science for centuries, combinatorics has now burgeoned into one of the fastest growing branches of mathematics . . . The mathematical world had been attracted by the success of algebra and analysis and only in recent years has it become clear . . . that combinatorics, the study of finite sets and finite structures, has its own problems and principles. These are independent of those in algebra and analysis but match them in difficulty, practical and theoretical interest, and beauty.
> László Lovász, *Combinatorial Problems and Exercises*

> In conclusion, we should like to caution the reader who might gather the idea that combinatorial theory is limited to the study of finite sets. An infinite class of finite sets is no longer a finite set, and infinity has a way of getting into the most finite of considerations. Nowhere more than in combinatorial theory do we see the fallacy of Kronecker's well-known saying that "God created the integers; everything else is man-made." A more accurate description might be: "God created infinity, and man, unable to understand infinity, had to invent finite sets." In the ever-present interaction of finite and infinite lies the fascination of all things combinatorial.
> Gian-Carlo Rota, *Discrete Thoughts*

The basic problem of enumerative combinatorics is that of determining the number of elements of a finite set. This enterprise is interesting, subtle, surprising, and very challenging. Enumeration is part of the larger discipline of combinatorics and has connections to algebra, analysis, topology, statistics, geometry, and a host of other areas in mathematics and science. Enumeration, and combinatorics in general, has a different feel than these other, more "traditional," areas of mathematics, a difference articulated by Timothy Gowers in the following passage.

> If the processes of abstraction and generalization, which are so important in mathematics, are of only limited use in combinatorics, then how can the subject be transmitted to future generations? One way of thinking about this question is to ask what the requirements of tomorrow's combinatorialists are likely to be . . . their priority is likely to be solving problems, so their interest in one of today's results will be closely related to whether, by understanding it, they will improve their own problem-solving ability. And this brings us straight to the heart of the matter. The important ideas of combinatorics do not usually appear in the form of precisely stated theorems, but more often as general principles of wide applicability.
> Timothy Gowers, *The Two Cultures of Mathematics*

Given this quality of combinatorial theory, our goals in this course are the following:

- To understand general principles of wide applicability for solving problems involving enumeration of finite sets, and
- To develop the ability to apply these principles to solve such problems.

We will develop a theory of enumeration based on common enumerative structures. Enumerative theory consists of understanding each of these common structures along with techniques for studying them. We will focus on the following ideas and principles.

(1) Basic structures of enumeration
   - Common enumerative families: binomial coefficients, Catalan numbers, Fibonacci numbers, Eulerian numbers, Bell numbers, Stirling numbers.
   - Lattice paths.
   - Compositions and partitions.
   - Recurrences.
   - Permutations and permutation statistics.
   - q-analogues.
   - The twelvefold way.

(2) Generating functions
   - Ordinary and exponential generating functions.
   - Tree enumeration and Cayley's theorem.
   - The exponential formula.
   - Lagrange inversion formula.

(3) Sieve Methods
   - The principle of inclusion-exclusion.
   - Unimodality and log-concavity of sequences.
   - The involution principle and sign-reversing involutions.
   - The Gessel-Viennot theorem.

(4) Partially Ordered Sets
   - Partially ordered sets (posets).
   - Lattices and their refinements.
   - The incidence algebra of a poset.
   - The zeta and Möbius functions for posets.
   - Computational methods for Möbius functions and the Möbius inversion formula.
   - Möbius algebras of lattices, Weisner's theorem, and the crosscut theorem.
   - Rank-selection, flag f- and h-vectors, and R-labelings.
   - Eulerian posets and duality.

3. TEXTS

Primary Text:

- *Enumerative Combinatorics, Volume 1, 2nd edition*, Richard Stanley. Preliminary version available at

  http://math.mit.edu/∼rstan/ec/ec1/.

Additional Resources:

- *The Two Cultures of Mathematics*, Timothy Gowers. Available electronically at

  www.dpmms.cam.ac.uk/∼wtg10/2cultures.pdf.

- *Chapter Six: Combinatorics*, Gian-Carlo Rota, from the collection of essays *Discrete Thoughts*. Available electronically from the UK Libraries website.

- *The Many Faces of Modern Combinatorics*, by Christian Lenart. Available electronically at
  http://www.albany.edu/∼lenart/articles/combin1.pdf.
- *A Course in Combinatorics*, Van Lint and Wilson, 2001.
- *A Course in Enumeration*, Martin Aigner, 2007.
- *A Walk Through Combinatorics: An Introduction to Enumeration and Graph Theory, 2nd edition*, by Miklos Bona, 2006.

4. Course Expectations

4.1. **Attendance.** You must be present at, prepared for, and engaged in class each day. If you need to miss class for some reason, please notify me ahead of time.

4.2. **Homework.**

- No late work will be accepted.
- Homework will be due on most class days. Homework will consist of problem sets and short essays. A selection of problems from each problem set will be graded. Short essays will receive credit for completion of assignment; credit will be deducted for particularly egregious violations of standard English grammar.
- Your homework must be typed in Latex. Latex is a fantastic system for typesetting mathematics and is the standard typesetting method for professional mathematicians. For more information, see http://www.latex-project.org.
- You may collaborate with your classmates in *developing ideas* regarding homework problems; however, do not let cooperation degenerate into one person solving the problem and other people copying their answers. While it is important to celebrate mathematics as a social and cultural endeavor, it is also important that *you work out the details* for solutions on your own. You must write up your own answers to all the questions. *For each homework problem, indicate in your solution the people you shared ideas with.*
- Searching the library or internet for solutions to problems is not allowed. The act of copying a written answer from another student and submitting it as your own will be considered cheating and will be dealt with according to the procedures referenced in Section 6.

4.3. **Exams.** There will be one take-home midterm exam and a take-home cumulative final. You are expected to work on these on your own and to follow all instructions regarding them. *You are not allowed to collaborate with other students on the exams.*

- The midterm will be distributed on Wednesday, March 2, and will be due in class on Monday, March 7.
- The final exam will be distributed on Friday, April 29, and will be due in my office by noon on Wednesday, May 4.

5. Course Grades

Your total grade will be determined by your homework and exams. The grading scale will be no stricter than the usual A>89.9, B>79.9, C>69.9, D>59.9, E otherwise, weighted as follows:

- Problem Sets: 30%
- Short Essays: 15%
- Midterm Exam: 25%
- Final Exam: 30%

6. Academic Integrity and Classroom Demeanor

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

http://www.uky.edu/USC/New/SenateRulesMain.htm

Turn off all cell phones, pagers, etc. prior to entering the classroom. ***You are not to use your cell phones, pagers, or other electronic devices during class.*** An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

7. Classroom and Learning Accommodations

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.