

MA 614 – Homework 10
Due Fri Mar 27

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Determine an expression for the number of labeled rooted trees such that each node is either a leaf (has no children) or has three children.
2. For n a positive integer, let a_n be the number of labeled rooted trees on $[n]$ such that there is a linear order on the set of children of each node and every node has an even number of children. Determine an expression for a_n .
3. Let $f(x)$ be a formal power series that satisfies $f(0) = 0$ and the cubic equation

$$(f(x))^3 - 2(f(x))^2 + f(x) - x = 0.$$

Find the coefficient of x^n in $f(x)$.

4. Assume $m \leq r \leq n$. Use an inclusion-exclusion argument to prove that

$$\sum_{k=0}^m (-1)^k \binom{m}{k} \binom{n-k}{r} = \binom{n-m}{r-m}.$$

5. This problem is a generalization of the inclusion-exclusion formula for finding the number of derangements in \mathfrak{S}_n . Fix $k \geq 1$. How many permutations of $[n]$ have no cycle of length k ? If $f_k(n)$ denotes this number, find $\lim_{n \rightarrow \infty} \frac{f_k(n)}{n!}$.