MA 614 – Homework 12
Due Friday, April 10

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. (a) Let P be a finite poset and $f : P \to P$ an order-preserving bijection. Show that f is an isomorphism.

   (b) Show that this can fail when P is an infinite poset.

2. (a) Prove that $B_n \cong (C_1)^n$.

   (b) Prove that if $n = p_1^{m_1} p_2^{m_2} \cdots p_j^{m_j}$ for distinct primes p_i and positive integers m_l, then $D_n \cong C_{m_1} \times C_{m_2} \times \cdots \times C_{m_j}$.

3. Prove that the rank generating function $F(P, q)$ for the power poset $C_n^{C_m}$ is the q-binomial coefficient $\begin{bmatrix} m+n+1 \\ n \end{bmatrix}_q$.

4. Let $\mathcal{C}$ be the set of all compositions of all positive integers. Define a partial ordering on $\mathcal{C}$ by letting τ cover $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_k)$ if τ can be obtained from σ either by adding 1 to a part, or adding 1 to a part and then splitting this part into two parts. More precisely, for some j we have either

$$\tau = (\sigma_1, \sigma_2, \ldots, \sigma_{j-1}, \sigma_j + 1, \sigma_{j+1}, \ldots, \sigma_k)$$

   or

$$\tau = (\sigma_1, \sigma_2, \ldots, \sigma_{j-1}, h, \sigma_j + 1 - h, \sigma_{j+1}, \ldots, \sigma_k)$$

   for some $1 \leq h \leq \sigma_j$.

   (a) For each $\sigma \in \mathcal{C}$, find a relationship between the number of saturated chains from the composition 1 (being the bottom element of $\mathcal{C}$) to σ and permutations with specified descent sets.

   (b) For fixed n, what is the total number of saturated chains that begin at 1 and end at a composition of n?