MA 614 – Homework 13
Due Friday, April 17

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Let n be an integer greater than or equal to 3. Determine the number of rank n distributive lattices such that each rank between 1 and $n-1$ (inclusive) has exactly two elements.

2. Let $k \in \mathbb{N}$. In a finite distributive lattice L, let P_k be the subposet of elements that cover exactly k elements, and let R_k be the subposet of elements that are covered by exactly k elements. Show that $P_k \cong R_k$ by describing an explicit isomorphism (use the structure of L as a finite distributive lattice to do this).

3. Let P be a finite poset with $\hat{0}$ and $\hat{1}$, and Möbius function μ. Show that

$$\sum_{s \le t} \mu(s,t) = 1$$

   where the sum ranges over all pairs $s \le t$ in P.

4. For the Boolean algebra B_n, prove that

$$\sum_{\hat{0} \le a \le b \le c \le d \le e \le \hat{1}} (\mu(\hat{0},a))^4 (\mu(a,b))^3 (\mu(b,c))^6 (\mu(c,d)) (\mu(d,e))^{10} (\mu(e,\hat{1}))^2 = 2^n\,.$$

   HINT: Since μ is always ± 1 in B_n, μ raised to an even power is equal to ζ.