MA 614 – Homework 2
Due Fri, Jan 23

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Explain why the identity
$$\sum_{m=k}^{l} \binom{m}{k} = \binom{l+1}{k+1}$$
is a consequence of induction and the Pascal recurrence for binomial coefficients. (HINT: Draw a picture illustrating which entries of Pascal's triangle are involved in the identity.)

2. Find a combinatorial proof of the following identity.
$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

3. Find a combinatorial proof of the identity
$$\binom{\binom{n}{2}}{2} = 3\binom{n}{4} + n\binom{n-1}{2}.$$

4. Find a combinatorial proof that
$$\left(\!\!\binom{n}{m}\!\!\right) = \left(\!\!\binom{m+1}{n-1}\!\!\right).$$

5. Find a simple "balls into boxes" (aka "dots and bars") proof that the total number of parts of all compositions of n is equal to $(n+1)2^{n-2}$. (The simplest argument expresses the answer as a sum of two terms).