# MA 614 – Homework 3
# Due Fri, Jan 23rd

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. (a) Use Taylor's formula (from Calc II) to derive the power series expansion

$$(1+x)^\lambda = \sum_{n\geq 0} \frac{\lambda\cdot(\lambda-1)\cdot(\lambda-2)\cdots(\lambda-n+1)}{n!}x^n$$

   where $\lambda$ is any complex number. Note that this is the Generalized Binomial Series.

   (b) Prove that a formal power series with coefficients in $\mathbb{C}$ has a multiplicative inverse if and only if the constant term is non-zero. (HINT: If $g$ is the inverse of $f$, then $f(x)g(x) = 1$. Multiply the series for $f$ by the series for $g$ and think about what this equality means for the coefficients.)

2. Let $f(x)$ be the ogf, i.e. the ordinary generating function, for the sequence $(a_n)_{n\geq 0}$. Express, using $f(x)$, the ogf for the sequence $a_0, a_0 + a_1, a_0 + a_1 + a_2, \ldots$, i.e. $\left(\sum_{i=0}^k a_i\right)_{k\geq 0}$.

3. For each positive integer $n$, express in terms of Fibonacci numbers the number of sequences $(a_1, a_2, \ldots, a_n)$ of 0's and 1's such that $a_1 \leq a_2 \geq a_3 \leq a_4 \geq a_5 \leq a_6 \cdots$.

4. Fix $k, n \in \mathbb{Z}_{\geq 1}$. Find a simple expression involving Fibonacci numbers for the number of sequences $(T_1, T_2, \ldots, T_k)$ of subsets $T_i$ of $[n] := \{1, 2, \ldots, n\}$ such that

$$T_1 \subseteq T_2 \supseteq T_3 \subseteq T_4 \supseteq T_5 \subseteq T_6 \supseteq \cdots.$$

5. Give a bijection between compositions of $n$ into 1's and 2's and compositions of $n+1$ into odd parts.