MA 614 – Homework 4
Due Friday, Feb 6

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Find the ogf's for the following sequences.
   (a) $(a_n)_{n\geq 0}$ where a_n is the number of compositions using parts from $\{1,3\}$.
   (b) $(b_n)_{n\geq 0}$ where b_n is the number of compositions using parts from $\{1,2,3\}$.
   (c) $(d_n)_{n\geq 0}$ where d_n is the number of compositions using parts from $\{1,2,3\}$ where the 2's are given one of five colors.

2. For $n \geq 0$, let a_n be the number of compositions of n using positive integers where every part is odd. (We choose by convention that $a_0 = 1$, since there is 1 empty composition).
   (a) Find the ogf for $(a_n)_{n\geq 0}$.
   (b) Using the resulting ogf, prove that $a_n = F_{n-1}$, the $(n-1)$-st Fibonacci number.
   (c) Where have you seen this result before?

3. For $n \geq 0$, let a_n be the number of compositions of n using positive integers where every part is greater than or equal to two. (We again choose by convention that $a_0 = 1$, since there is 1 empty composition).
   (a) Find the ogf for $(a_n)_{n\geq 0}$.
   (b) Using the resulting ogf, express a_n in terms of Fibonacci numbers.

4. Use Euler's generating function identity for $p(n)$ to find an expression for the number of partitions of n with no part equal to 1 or 2 in terms of values of the partition function $p(n)$.

5. Let $p_k(n)$ denote the number of partitions of n with exactly k parts. Let $q_k(n)$ denote the number of partitions of n into exactly k distinct parts. For example, $q_3(8) = 2$, given by $(5,2,1)$ and $(4,3,1)$. Give a combinatorial proof that

$$q_k\left(n + \binom{k}{2}\right) = p_k(n)\,.$$