MA 614 – Homework 5
Due Friday, Feb 13

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Consider the sequence defined by $a_n = \sum_c 2^{c_1-1} \cdot 2^{c_2-1} \cdots 2^{c_k-1}$ where $c = (c_1, c_2, \ldots, c_k)$ ranges over all compositions of n.
   (a) Find the generating function $\sum_{n\geq 0} a_n x^n$.
   (b) Use the generating function to find an explicit formula for a_n.
2. Consider the sequence defined by $a_n = \sum_c c_1 \cdot c_2 \cdots c_k$ where $c = (c_1, c_2, \ldots, c_k)$ ranges over all compositions of n. Find the generating function $\sum_{n\geq 0} a_n x^n$. (HINT: Consider the effect of applying the operator $x \cdot \frac{d}{dx}$ to the geometric series.)
3. Let $S(n, k)$ denote the Stirling numbers of the second kind. For $n \geq 1$, prove directly the following (mentioned at the top of page 74 in EC1):
   (a) $S(n, 2) = 2^{n-1} - 1$
   (b) $S(n, n-1) = \binom{n}{2}$
   (c) $S(n, n-2) = \binom{n}{3} + 3\binom{n}{4}$
4. On page 74 of EC1, there are six identities related to Stirling and Bell numbers, labeled (1.94a)-(1.94f). The proofs given in the text are very succinct and omit many details. Provide clear, detailed proofs for each of these six identities, following the arguments given by Stanley.