MA 614 – Homework 6
Due Friday, Feb 20

Your answers should be detailed explanations in quality mathematical English. You must type your homework in LaTeX.

1. Direct from the definitions (without using the ogf or any recurrence), compute the following.
   (a) $c(n, n-1)$
   (b) $c(n, n-2)$
   (c) $c(n, n-3)$
2. Let $f(n)$ be the number of ways to choose a subset $S \subseteq [n]$ and a permutation $w \in \mathfrak{S}_n$ such that $w(i) \notin S$ whenever $i \in S$. Show that $f(n) = F_n n!$, where F_n is the nth Fibonacci number.
3. Let $f_k(n)$ denote the number of permutations in $\mathfrak{S}_n$ with k inversions. Show combinatorially that for $n \geq k$,
$$f_k(n+1) = f_k(n) + f_{k-1}(n+1)\,.$$
Using this recurrence, prove that for $n \geq k$, the sequence $f_k(n)$ is given by a polynomial in n of degree k and leading coefficient $1/k!$.
4. Let $A_n(x)$ be the Eulerian polynomial.
   (a) Give a combinatorial proof that $\frac{1}{2}A_n(2)$ is equal to the number of *ordered* set partitions of an n-element set, i.e. partitions whose blocks are linearly ordered.
   (b) More generally, show that
$$\frac{A_n(x)}{x} = \sum_{k=0}^{n-1} (n-k)!S(n, n-k)(x-1)^k\,.$$
   Note that $(n-k)!S(n, n-k)$ is the number of ordered partitions of an n-set into $n-k$ blocks.