# Personal, Expository, Critical, and Creative: Using Writing in Mathematics Courses

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**Abstract**

This article provides a framework for creating and using writing assignments based on four types of writing: personal, expository, critical, and creative. This framework includes specific areas of student growth affected by these writing styles. Illustrative sample assignments are given throughout for each type of writing and various combinations thereof. Also discussed are the assessment of mathematical writing and suggestions for beginning users of writing in mathematics courses.

## 1 Introduction: Why Writing?

This article provides a conceptual framework for creating and using writing assignments based on four types of writing: personal, expository, critical, and creative. One of the strengths of this framework is that each type of writing is associated with specific learning outcomes, making each writing type a distinct tool with a clear purpose. The inclusion of learning outcomes associated with each type of writing makes this framework amenable to potential study by mathematics education researchers. At present, the evidence supporting this framework consists of informal data, primarily my own anecdotal experiences along with excerpts from the work of my students; without fail, their insights and mathematical yawps surprise and inspire me. (In [14], Francis Su defines a mathematical yawp as "that expression of surprise or delight at discovering the beauty of a mathematical idea or argument.")

In addition to being applicable, adaptable, and testable, this four-type framework provides a context for further investigation by teachers of the use of writing in mathematics courses. The general field of composition pedagogy is broad and multifaceted, with multiple (often conflicting) approaches to the theory of composition [8, 16]. One of these composition theories, the Writing

Across the Curriculum movement, has been around for several decades and has had an impact on undergraduate mathematics education, along with other disciplines [3]. Mathematicians have also been directly involved in promoting the use of student writing, and they have produced a wealth of ideas, assignments, and advice for interested teachers [5, 7, 13]. Further, writing assignments complement and extend many active learning techniques, such as Inquiry-Based Learning, which are of increasing interest to mathematicians, scientists, and engineers [6, 11, 12].

In his decade-long study of the practices of highly-effective college teachers, Ken Bain found that regardless of their discipline, such teachers establish in their courses a *natural critical learning environment*, described by Bain [1] as follows.

> In [a "natural critical learning environment"], people learn by confronting intriguing, beautiful, or important problems, authentic tasks that will challenge them to grapple with ideas, rethink their assumptions, and examine their mental models of reality. These are challenging yet supportive conditions in which learners feel a sense of control over their education; work collaboratively with others; believe that their work will be considered fairly and honestly; and try, fail, and receive feedback from expert learners in advance of and separate from any summative judgment of their effort.

While problem solving and theorem proving are typically the primary focus of mathematics courses, I believe that to create this type of learning environment in a mathematics course we must have a vision of student mathematical development extending beyond these two activities. To illustrate this point, it is helpful to consider what a math course often feels like to our students (even to many of our "good students").

At the beginning of a course, the instructor tells the students several things they are expected to understand. The instructor then gives a homework assignment on those topics; if the students do not complete the homework correctly (regardless of their effort and/or interactions with the instructor), they are penalized. This process continues until an examination is given, at which point students are given a variety of problems to solve or theorems to prove; if they make mistakes, they are again penalized. This cycle iterates until the end of the semester or quarter, at which point the students are given a final exam (typically cumulative), and if they make mistakes on this exam, they are again penalized. At this point the course ends, and many (most?) students promptly forget all the things they have been required to learn for homework and exams.

For most students, this has been their mathematical experience starting from a very young age. They don't really *know* what it is that we want them to *do* — how we want them to grow — outside of this structure. To engage students in a natural critical learning environment, to help them learn to view mathematics and mathematical activity in a more mature way, we must find ways to directly affect their mathematical habits. As David Bressoud points out [4]: "Assessment is the carrot and stick that you can use to shape student attitudes and study habits and to communicate what you want students to learn from your course." Thus, if we want to change outcomes for our students and our courses, we must reconsider what our students are doing and what they are being asked to do.

The effects of mathematics courses of the kind just described are not only faced by students in service courses, or restricted to under-performing students in technical majors. Most of my colleagues are horrified when I tell them about what I and many of my mathematically-inclined friends did not learn during our K-12 and undergraduate experiences. I'm not talking about subtle points here; I'm talking about fundamental ideas such as the answers to the following questions.

- Why is the equation of the unit circle $x^2 + y^2 = 1$?
- Why is the graph of the sine curve periodic? Why does it have the shape it does?
- Why is $\pi$ approximately 3.14?
- Why is the Pythagorean theorem true?

It is a fascinating exercise to ask students these questions, and to experience the general lack of coherent answers coupled with the guilt and embarrassment felt by students regarding their lack of understanding. Again, I'm not only talking about students in college algebra or business calculus; discussing the answers to these questions is often a door-opening experience for math, math education, and science majors in upper-division courses.

One of the reasons so many students can "succeed" in our education system without understanding many key mathematical ideas is that fundamental questions such as those given above are difficult to administer, and their answers are even more difficult to assess and respond to. As a result, they are rarely asked, and students can get by knowing only a disjointed collection of "facts," without having any sense of why the facts are true. It is in this context that writing can come to our aid, allowing students to participate in genuine dialog with their instructors and peers, helping teachers create natural critical learning environments in which students develop profound understanding of mathematical ideas. At their best, writing assignments can help students develop a sense of committed ownership of mathematics, as shown in the following passage.

> The first and possibly most significant realization that I made in this class was the one that math is debatable and changeable. Throughout my education, I have always learned mathematical methods as if they were set-in-stone laws that couldn't be questioned. Learning that math is constantly changing, evolving, or being proven incorrect was a profound moment for me. I have enjoyed being able to have an opinion and discuss math as if we were discussing a novel and that is something I will always carry with me. I have noticed that since I came to this realization, I have had several conversations with people who have talked about how math is unlike other subjects because it just IS. Of course, I always make a feeble attempt to correct their judgment, but they usually stop listening or get overwhelmed once I start ACTUALLY talking about math.
>
> Andrea Meadors, student

## 2 Student Development Through Writing

There are five specific areas of student growth that I consider when designing writing assignments, areas adapted from and inspired by Donald Murray's discussion [9] of the effect the process of revision can have on writers. Through writing, students can:

- change how they feel about what they know;
- discover that they understand more (or less) than they thought;
- develop perspectives on the work of others;
- extend what they understand; and
- deepen their commitment to mathematics.

In the following subsections, I discuss how the four writing types in this framework can contribute to these areas of student growth. In addition to discussing the effect writing can have on student growth, I discuss writing assignments I've used in my courses. These assignments should be taken as inspiration rather than prescription; each teacher will need to change and adapt such prompts, and create new assignments, to fit the needs of their students.

### 2.1 Personal Writing

Through personal writing, students can change how they feel about what they know. Directly confronting the issue of emotional engagement in the classroom does not require that we commit a great deal of time to this, or that we become some kind of therapists, but it does require that we ask students to write about their personal mathematical experiences in meaningful ways. Whenever possible, I like to have my first homework assignment in a course be a mathematical autobiography, with the purpose of creating a student-centered focus for the course.

Various resources regarding writing in mathematics courses suggest having students write an autobiography at some point. In my experience, asking students to write a generic "autobiography" is too broad, and makes students feel like they have to discuss their entire mathematical life in a page (which is impossible). Instead, I prefer the following assignment.

> Imagine that you have written a book-length autobiography about your mathematical experiences. Type a passage, thought of as a quote from your autobiography, that reveals something significant about you mathematically. Be as creative as you like, but try to keep it around a page or less.

Allowing students to quote from an imagined source gives them more flexibility in choosing something mathematically meaningful to write about, without worrying about discussing everything they've ever done in math. This serves as the first assignment of a course, due on the second day of class. The grade for the assignment is based only on completion – at this point, I don't care how students write or how well they write, I just want to start the class with a focus on the students and their experiences (this perspective is similar to themes in the theory of *expressivism* in composition pedagogy [16]). I use twenty minutes at the beginning of the second day of class to have each student say their name and read one or two sentences from their autobiographical excerpt.

If possible, I also have the students sit in a semicircle or circle so that they can see each other. For classes of under thirty-five students, this seating arrangement should be viewed as an integral part of the autobiography assignment, a part that continues through the course, as it encourages students to view each other as peers rather than as strangers in a room listening to a lecture. In my end-of-course evaluations, this non-standard physical classroom layout has consistently inspired positive comments from students, with two representative examples being: "when we stepped into class on the first day and the chairs got pushed back into a large circle, I knew right away that I was going to like the class" and "day one of classes – syllabus day – I attended class expecting the same spiel, only to be greeted by something I completely did not expect: a circle, yes a circle, of desks all facing inward. . . I would quickly come to admire the design." Such an arrangement would obviously be a challenge in large lectures, where I have not yet implemented this assignment.

Once a student-centered tone for the course has been established, this tone can be nurtured through the use of homework cover letters or other similar assignments. A homework cover letter is a one- or two-paragraph response, handwritten, to a question or prompt. The homework cover letter is again not graded, but I will not grade homework without a completed cover letter. Here are two sample questions I have used.

> What has been the hardest topic for you so far in the course? Why?

> What was one of your favorite homework problems from this set? Why? What was one of your least favorite? Why? Be specific in your reasons!

There is one caveat that must be made about homework cover letters. I have found that as a stand-alone writing tool, cover letters are not particularly effective at increasing student engagement. Students who are not engaged in a course don't take the cover letter seriously, while the students who are engaged in the course provide very interesting feedback. However, if the student responses are actively used in some way in the course, or are used in conjunction with other activities to enhance student engagement, they can be effective. One example of such use might be using student responses as discussion prompts for group work during class.

Often students change how they feel about mathematics through an experience that isn't related to a writing assignment; subsequent writing then provides students an opportunity to reflect on this change and their new perspective. Thus, we should think of writing as a tool for both changing how students feel about mathematics and for reflecting on their changing attitudes. With this in mind, here is an assignment that I like to give as the final assignment of a course. This is again graded based only on completion.

> Type a 2.5–3 page short essay (Times New Roman, double-spaced, 12 point font, 1 inch margins) in response to the following prompt.
>
> - What were *six* of the most important discoveries or realizations you made in this class? In other words, what are you taking away from this class that you think might stick with you over time and/or influence you in the future? What have you experienced that might have a long-term effect on you intellectually or personally?
>
> These can include things you had not realized about mathematics or society, specific homework problems or theorems from the readings, discussions with other students or the professor, connections to other courses, etc. These can be things that made sense to you, or topics where you were confused, points that you agreed/disagreed with in the readings or class discussions, issues that arose while working on your course project, etc. Explain why these six discoveries or realizations are important to you.
>
> *Please make an effort to include a combination of observations directly dealing with mathematical content and observations that are more general.*

I like to think of this assignment as a complement to the autobiographical excerpt. We start the class with the students thinking about their mathematical past, then we collectively engage in new and interesting mathematical explorations. We end the class by returning to a reflective mode, where we contemplate our experiences and our mathematical future; the following quote illustrates the kind of emotional development in students that writing can uncover.

> I am consciously aware now of something I already knew but hadn't yet articulated to myself: the further one gets in science or math, the more one develops strong emotional impressions about 'facts' (or equations or matrices)... a matrix that can be represented as another similar matrix that in turn makes computations easy and allows for corresponding real-world measurements is just downright useful, and therefore 'good.' It is not really a value judgment *per se* or a loss of objectivity, but it is an honest recognition that some matrices are better for some things than others.
>
> Jane Dunford-Shore, student

## 2.2 Expository Writing

Through expository writing, students can discover that they understand more (or less) than they thought. Professional mathematicians know that writing about the work of others is an excellent way to learn mathematics; the same is true for students. While essays are what likely come to mind for most people when expository writing is mentioned, there are many ways to incorporate expository writing into courses. Here is a homework problem I like to give, which I've used in service courses, major courses, and graduate courses.

> If one of your fellow students was confused about topic X, what specific example or examples would you use to help them get a better idea what is going on? Why? Your response should be typed, one page, double-spaced.

I grade responses to this prompt using the grading rubric discussed in Section 3. I like to require a typed response to this prompt rather than a handwritten one. Students often use too many symbols when responding to writing prompts in handwritten work, interfering with the articulation of their understanding of the symbols. Typed short responses encourage students to use words rather than symbols, since adding symbols is less convenient in word-processing programs undergraduates are likely to use, e.g. Word.

While this prompt is simple, it can be very effective at revealing student concept images (as compared to concept definitions; for more on the useful idea of concept images versus concept definitions, see the work of Tall and Vinner [15]). For example, when teaching a sophomore/junior level linear algebra course, I included this question where topic X was linear transformations. One of the top students in my class responded with the example of the Fahrenheit to Celsius conversion, which is affine rather than linear. After I returned homework, he asked me about the example, and it became clear that he had not yet internalized the idea that linear transformations fix the origin. I definitely would not have caught this misunderstanding from this student, as his problem sets were otherwise exemplary.

Another good type of expository writing for students is the biographical essay. I have only assigned a pure biographical essay once, and subsequently changed the assignment. I have found it helpful to require that in essays with a biographical component, students include a mathematical component in addition to the historical component. If such an essay includes a required mathematical component, they can be graded using the standard rubric in Section 3.

One last example of an expository assignment is to have a component of homework and/or exams where students are asked to write about the material under consideration. The following prompt lends itself to many variations and extensions.

> What were the main mathematical objects, problem types, and solution techniques covered on this homework/exam? How are they related to each other? What is the reason for developing and understanding these concepts, problem types, and solution techniques?

While I haven't used this exact prompt in courses, I have often asked students to compare and contrast multiple proofs of a theorem, or to provide condensed outlines of proofs. Student responses to such prompts can be graded using the standard rubric. If used on an exam, similar prompts should be included on prior homework assignments so that students aren't surprised by what is being asked of them.

If using a prompt of this type, where the student undertakes expository writing about the current course material, it is important to separately ask in the prompt for students to address the objects under consideration, the problem specification considered (i.e. compute the slope of the tangent line, or find the "best" solution possible to an unsolvable linear system), and related problem solving techniques. Students are often completely focused on problem solving, without giving attention to the issues of object definition and problem specification (which are key ingredients to understanding solution methods!). Expository writing can help students articulate and clarify these and other meta-mathematical paradigms, as the following quote shows.

> [B]efore this class I had never even considered the notion that "squaring" a number was simply the concept of drawing a square of a given length. Growing up, I guess I always lived in the algebraic world and never really ventured into geometry much. However, notions like this make demonstrating and proving concepts much more intuitive sometimes. Euclid's proof of the Pythagorean Theorem is the perfect example. Previously, I just would not have made the connection that $a^2 + b^2 = c^2$ is simply saying the sum of the squares drawn on the two legs of the triangle is the square drawn down the hypotenuse. For me, there was always this disconnect between algebra and geometry, but now I realize how using geometric representations can be very helpful in solving algebraic problems.
>
> Matthew Seabold, student

## 2.3 Critical Writing

Through critical writing, students can develop perspectives on the work of others. I consider the act of developing perspectives on the work of others as akin to the "critical thinking" that is often advocated for our students, but more refined. For example, in literary critical theory, there are many different schools of literary analysis: formalism, historicism, feminist critique, reader response theory, deconstructionism, etc. Each of these modes of critical analysis brings a different perspective to the work under consideration, providing a clear set of tools for critical analysis. Mathematicians do not generally separate the work of "critics," whose primary job is to comment on the work of others, with "creators," whose primary function is to produce new works, as distinctly as other disciplines, and we have not developed a clearly-articulated, diverse set of approaches to critical analysis.

I think it is generally a positive thing that mathematicians function simultaneously as creators and critics. However, I don't believe that students typically understand or appreciate the difference between these activities, or see these activities clearly manifested in their courses. Problem solving and theorem proving, even in the context of homework assignments that re-derive established results, are creative work. Without asking our students to explicitly take on the task of critical analysis, we leave it to chance whether or not they develop perspectives on the mathematics they are learning about through creative work, and we leave them without clear tools for critical analysis.

One technique of critical analysis we can teach students is to make writing a part of the act of reading, and vice versa. A version of this occurs when mathematicians say that to read mathematics one needs to have pencil and paper at hand; at a broader level, this is a core tenet of many scholars involved in composition pedagogy (though this is also a subject of serious debate). For example, David Bartholomae and Anthony Petrosky write the following in *Ways of Reading* [2], a frequently-used first-year composition textbook.

> Strong readers. . . remake what they have read to serve their own ends, putting things together, figuring out how ideas and examples relate, explaining as best they can material that is difficult or problematic. . . At these moments, it is hard to distinguish the act of reading from the act of writing.

With this in mind, one of my favorite assignments to give to students is the following.

> Write a three page critical review of Object X (where Object X can be an assigned reading, a youtube video, a wikipedia page, etc). Imagine that you are writing your review for a journal for undergraduates in mathematics and the sciences. You must address the mathematical depth and mathematical style of Object X in addition to other topics.

I grade this assignment using the grading rubric in Section 3, and typically require a typed response, double-spaced, of between two and four pages depending on what Object X is. I find that having students write critical reviews of masterful expository work is sometimes less helpful than having them write reviews of texts or media that have some proverbial warts. A warning: I have assigned critical reviews of texts that I thought were wonderful, but that did not stand up to student scrutiny. This can be one of those wonderful assignments where teachers gain insight from students.

Another example of critical analysis is to ask students to compare two approaches to the same mathematical idea. For example, plane geometry can be introduced axiomatically in the style of Euclid, or through coordinate methods assuming some axioms about the reals. A variant on a problem I have given in the past, and which can be adapted to many different topics, is the following.

> Right triangles have different definitions in Euclidean plane geometry (neighboring angles on a line are equal) and in coordinate geometry (dot product of two sides is zero, assuming the origin at a corner). How are these definitions related? Why do they both describe the same type of object? Which of these definitions is more intuitive? If you were explaining right triangles for the first time to someone, which definition would you start with? Why?

This assignment would be typed, 3-4 pages long, double-spaced, and would be graded with the rubric in Section 3.

Sometimes writing prompts arise from the work of students. In the passage below, one of my students writes about his experience learning Newton's approach to power series; this passage came from an end-of-semester reflection, not an assignment that was explicitly critical in nature. However, one can easily find the genesis of a written prompt for future classes in the passage below.

> . . . what surprised me is that what we have learned in class wasn't even necessarily how mathematical ideas progressed, even when Europeans were the "inventors." For example, Newton's approach of limits didn't really seem to involve use of Riemann sums like we've been taught. While the same geometric approach to the idea applies, he instead analyzed these ideas by use of series expansions and his binomial theorem. This is mainly due to the fact that he "knew" the integration of polynomials and thus by finding the appropriate series he could accurately approximate integrals to obtain their values. . . It is unfortunate however that this is not presented [earlier]. I understand why it's not presented in [Calculus I], because their concern is with integration by use of the antiderivative and so the details of how Newton used them aren't entirely appropriate. However, if they presented it as *Journey Through Genius* did this would have made me have an actual appreciation and understanding for the use of series expansions.
>
> Rob Milburn, student

## 2.4 Creative Writing

Through creative writing, students can extend what they understand. Problem generation, i.e. the act of creating interesting yet viable questions and recognizing or developing tools to investigate these problems, is a critical skill in mathematics that is almost never addressed in coursework. However, the skill of problem generation, and related applied issues such as mathematical modeling, can serve students well in the long run. I feel that mathematical creative writing is far underutilized in our courses as a means of helping students develop tools for generating problems and independently extending what they understand.

The creative writing to which I am referring is not fiction or poetry; rather, it is writing that is intimately intertwined with being mathematically creative. This section will be brief, as it discusses the type of writing with which I have the least direct classroom experience. However, I believe creative writing has great potential for helping students develop mathematically, including in the areas of problem solving and theorem proving.

The best example I have of this kind of writing is inspired by the article of Michael Orrison [10] regarding a multi-day assignment he gives students in his discrete mathematics course. My variant of the assignment is the following.

> Define a measure of complexity of a graph. Determine various properties of this measure of complexity. Write a 3-4 page paper that motivates your measure of complexity and establishes its properties.

We often view this kind of mathematical creative writing as an activity done at the graduate level, when students author or co-author research papers, theses, and dissertations (though we don't usually refer to these works as creative writing). At many institutions, undergraduates either can or must write a senior capstone paper, and these can serve as vehicles for creative writing as well.

Through the use of creative writing projects on fundamental topics in mathematics, students can simultaneously experience the thrill of discovery, problem creation, and contextualization, while also learning mathematics that is widely applicable. I believe that introducing mathematical creative writing in our curricula in effective ways is a serious challenge for those of us using writing in math courses. It is a crucial task that deserves our serious thought.

## 2.5 On Committing to Mathematics

In the Perry model for intellectual development [1, pgs 42-43] and its many variants, the highest level of undergraduate intellectual development is a deep commitment to a discipline. As is hopefully clear, all four types of writing described here can help students commit to mathematics more deeply. In this subsection, I want to focus discussion on the role long-term course projects can play in this regard.

First, a digression. As an undergraduate, I was a double-major in Mathematics and English. In my English courses, students were expected to read a variety of texts, then engage in discussions in class. In response to these texts, we were expected to write four or five papers during the course of the semester, plus revisions. At the end of the semester, we compiled our work into a portfolio and wrote a cover letter discussing the work and our personal intellectual development. This was not perfect, in that some students didn't read the texts, offered weak occasional comments in class, wrote poor essays, etc. Yet for many students, this structure provided an opportunity for them to sincerely reflect on their own learning and growth, and to receive credit and acknowledgment for this act.

My mathematical teaching has been strongly influenced by these experiences with upper-division humanities courses. Through my experiences as a student and teacher, I have come to believe that when students are assessed over a longer period of time, with the freedom to initially make mistakes without severe penalty, they generally develop a deeper commitment to mathematics than otherwise. Thus, I believe that extended course projects should be expected of mathematics students at the upper-division level.

My experience assigning course projects has primarily been in junior-senior level History of Mathematics courses. The book of Meier and Rishel [7] includes a general discussion of the use of course projects in upper-division courses such as modern algebra, advanced calculus, operations research, etc. Here is the prompt for the course projects I assign in History of Mathematics; these are graded and peer-reviewed using the rubric in Section 3.

> Students must write a 16-20 page written project related to the history of mathematics, double-spaced, Times New Roman, 12 point font, 1 inch margins. While suggested topics are provided, students may pursue topics from "off-list." The paper must be directed at both a general university audience and their peers in the course. Seven

> bibliographic items required, but only two of these may be web-published content (i.e. accessing a book or journal article electronically is not considered web-published content, while wikipedia, mathworld, etc are web-published). A substantial revision process is required, with instructor and peer-review on the first version of the project; the first version of the project is a complete, polished project, not a rough draft.

During a 14-week course, I require students to turn in a one-page, ungraded project proposal during week five. First versions of projects are due 10 days prior to the due date for midterm grades, to allow me time to grade. The final, revised versions of course projects are due two weeks prior to the start of final exams, giving me time to grade and getting the projects finished prior to when students begin studying for finals in other courses. I give extensive written feedback on the first versions, and almost no feedback on the final versions aside from the grade. An important point is that I require students to turn in their first versions with their final versions; by making extensive written comments on the first version, I am able to see my prior comments when judging the student score on "depth of revision" in the grading rubric.

Extended course projects give students an opportunity to take ownership of a mathematical topic and to become a specialist in the area among their peers. To paraphrase one of my students, "the project is great because it gives you a niche topic which is your own, which no one else knows about." Generally, most of my students are very worried about their course project. Yet for most students, the main challenge with the final version is to keep their project *under* the twenty-page limit! Producing a course project, one which incorporates the process of revision and provides a record of this process, gives students tangible evidence of their mathematical growth that is hard to match with other types of work.

In addition to course projects, there are other ways to get students to reflect on their body of work in the course, hopefully leading to a greater commitment to mathematics. In one of my sophomore/junior level matrix algebra courses, I required students to keep all their homework and exams. At the end of the semester, the students were required to turn in a portfolio containing these items and a 2-3 page cover letter reflecting on their growth through the semester, their greatest challenges in the course, their favorite and least favorite topics, etc, with specific references to homework and exam problems in the portfolio. The cover letter (graded only by completion) required students to think about their work in a manner that studying for the final exam (even though it was cumulative) did not engender.

The following quote illustrates how a course design that engages and empowers students, providing them opportunities to develop through multiple types of writing, can have an incredible effect on student commitment to mathematics.

> I don't actually hate math. . . I will never forget the moment I walked out of class wanting to figure out the different proofs on the infinitude of primes worksheet. I didn't need to figure them out – there wasn't going to be a test over the material and the worksheet didn't even count for a grade. But for some reason, I actually wanted to sit down

outside of a classroom setting and think about math for fun... This is all coming from somebody who "hated" math in high school. On top of all that, I realized I might actually like math when I figured out some of the more difficult problems involved in my course project and was overwhelmed with excitement.

Nicole Schladt, student

## 3 Assessing Mathematical Writing

Whenever I talk to teachers about using writing assignments, the issue of grading arises almost immediately. Many resources for using writing in math courses describe the use of checklists to assess writing, which is helpful for writing based on problem solving but which I find less useful in the context of the assignments I describe here. Instead of a checklist, I have developed a grading rubric for mathematical writing. Student peer review is also done using this rubric, with the intention of getting students to actively think about these aspects of their own writing through the evaluation of others.

The rubric presented here has not been developed from scratch. I looked at many of the grading rubrics for writing available at various university writing centers, and adapted what I felt were the most relevant aspects of them for mathematics. The rubric contains six areas for assessment. When used for short essays, only the first five grading criteria are applied. When used for course projects, the first version of the project receives a score of 0 through 10 for each of the first five criteria. The final version of the project receives a score of 0 through 10 for each of the first five criteria and 0 through 25 for the sixth.

The grading rubric is presented to students in the following form.

**Writing Style:** Score: First: ________ Final: ________

A 10 paper is eloquent and effective, with varied sentence structures, good rhythm, fluid transitions, and a distinct voice. A 7 paper is coherent and appropriate, but uneventful and uninspiring. A 3 paper often contains sentences that are not comprehensible and alienating to the reader.

**Arrangement and Development:** Score: First: ________ Final: ________

A 10 paper guides the reader through the text with organizational clarity and ingenuity, providing the reader with the information that is needed at each moment. A 7 paper does not go out of its way to help readers, but is reasonably well structured and logically sound. A 3 paper is skimpy or bloated, with haphazard organization, regular disregard for logic, and little consideration for the reader.

**Editing and Conventions:** Score: First: ________ Final: ________

A 10 paper demonstrates maturity with regard to grammar, syntax, word choice, and attribution of sources. A 7 paper features distracting errors in phrasing, punctuation, citation, etc. A 3 paper has regular or repeated problems with these features that impedes reader comprehension.

**Mathematical Depth:** Score: First: _________ Final: _________

A 10 paper demonstrates a sure grasp of mathematical content, providing interesting mathematics with insightful connections from the material to other areas of mathematics and science. A 7 paper responds appropriately to the assignment, but either does not contain deep mathematical content or does not effectively communicate the worth of the material. A 3 paper exhibits mathematical errors that prevent understanding by the reader.

**Mathematical Style:** Score: First: _________ Final: _________

A 10 paper illustrates the mathematics under discussion with clear proofs, illuminating examples, or a combination thereof, and stimulates the intellect. A 7 paper provides too many or too few details in proofs and/or unenlightening examples. A 3 paper is often incomprehensible, even if mathematically correct.

**Depth of Revision:** Score: First: _________ Final: _________

A 25 paper demonstrates a mature and holistic approach to the revision process and has been revised in areas that were weaknesses and also in areas that were prior strengths. An 18 paper contains revisions, but only to specific areas of great weakness and without regard for overall improvement to the paper. A 12 paper contains only minor revisions and demonstrates a lack of effort on the part of the author.

**Instructor Comments/Peer Editing Feedback**

(Sizable blank space left for comments.)

When appropriate, I give scores using half-points, e.g. 8.5/10. Obviously a normalization could be used to place the score out of twenty, but for me it is simpler to assign fractional grades from ten points. Also, while one could attempt to outline finer gradations regarding the scoring, I find that maintaining some flexibility is helpful.

Students typically have no questions about the first three assessment areas, but the areas of mathematical depth and mathematical style take getting used to. Mathematicians are familiar with the distinction between these: most of us have read papers with amazing results that were horribly written, and we have likely also read papers with clear examples and precisely stated theorems that were utterly trivial. I find that the easiest way to help students think about the difference between mathematical depth and mathematical style is to illustrate it through readings, and by using the rubric on their short essays early in the course. While I have found that this rubric is suitable for assessment of the assignments I grade, one should always be willing to adjust it as the need arises.

## 4 Suggestions

In this final section, I provide a few suggestions for teachers using writing in their courses for the first time.

- *Start slowly!* If you have never used writing in your courses before, don't start by introducing a course project. Autobiographies, reflective essays, and short critical essays are much easier assignments to incorporate into a course, and to grade.

- *Be clear about your expectations with your students.* Students are usually shocked when asked to write in their math class. From their responses, it sometimes seems like they've never been asked to write before in any class. It is good to ease them into it: start with an ungraded writing assignment, then shorter works. Make sure the students see the grading rubric early, and have plenty of time to ask questions about it before their first assignment is due.

- *Plagiarism is an issue.* If you are going to use writing in your course, particularly with course projects, plagiarism is a major issue. Many students do not know that copying text directly from websites or wikipedia constitutes plagiarism (such sources are often viewed as "public knowledge"). Find out what your college or university policy is regarding plagiarism, include it in your syllabus, and discuss it with your students.

- *Make friends with the writing faculty.* If you are serious about using writing in your course, send an email to a faculty member in the humanities asking for advice. Our colleagues who teach writing have experience with both general writing-related issues and issues that might be idiosyncratic to specific institutions.

- *Use fixed texts to prompt writing assignments.* A good first writing assignment for students is to respond to a fixed text. This gives students a concrete object to refer to, serving as a catalyst for their writing. Remember: a fixed text does not have to be their textbook. It could be a website, video, or even their own homework or exams!

- *Find out more about the mathematical writing community.* There are many mathematicians interested in using writing in math courses. Sectional and national meetings of the MAA regularly have panel discussions or paper sessions on this topic. It is inspirational to hear what others are doing, so if you can find a session to attend, it's worth going.

A final thought: I love hearing from teachers who are interested in using writing in mathematics courses, or who are already doing so. If you put some of the ideas from this article into practice, please let me know about your experiences and those of your students. Good luck!

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