Contents

Practice Final Exams

Practice Final Exam #1.

(100 points in all; time = 3 hours, checking your work carefully)

1. (15 points) Evaluate: (a) $\int_0^3 \sqrt[3]{1-3x}\,dx$ (b) $\int \sin\left(\frac{\pi}{10}(t-1)\right)\cos\left(\frac{\pi}{10}(t-1)\right)\,dt$.

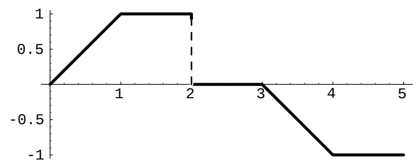
2. (15 points) A light is at the top of a pole 14 meters high. A ball is dropped from the same height from a point 7 meters from the light. How fast is the shadow of the ball moving along the ground 0.8 sec later? (Assume that the ball falls a distance $s = 4.9t^2$ meters in t seconds.)

3. (15 points) At time $t = 0$ an object is at rest. It then accelerates according to the graph at the right, where the acceleration $a(t)$ is measured in m/sec^2.

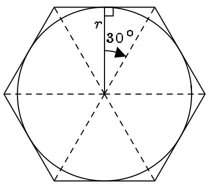
(a) Sketch the graph of the velocity function $v(t)$ during these 5 seconds.

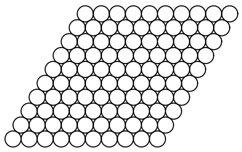
(b) Find a formula for $v(t)$ for t between 0 and 1, for t between 1 and 2, and for t between 2 and 3.

(c) How far does the object travel during the first 3 seconds?



4. (15 points) Cylindrical cans with circular tops and bottoms are to be manufactured to contain a given volume. There is no waste involved in cutting the tin that goes into the vertical side of the can. But the two circular end pieces are cut from a sheet, as shown at the bottom of the page. If the circles for the tops and bottoms of the cans are fit as snugly as possible, each uses up a hexagonal area of tin, as shown in the picture above. First find the area of one of the hexagons in terms of r. (Hint: It consists of 6 equal equilateral triangles.) Now find the ratio of height to radius for the most economical cans. In other words, minimize the total area of tin needed to make a can.





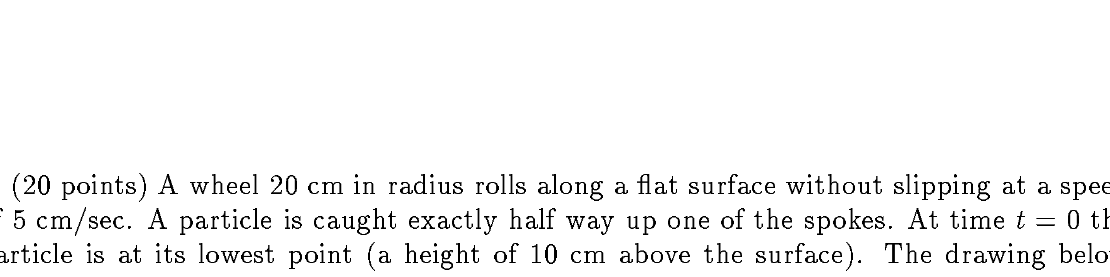
5. (20 points) A 4-centimeter rod is attached at one end A to a point on a wheel of radius 2 cm. The other end B is free to move back and forth along a horizontal bar that goes through the center of the wheel. At time $t = 0$ the rod is situated as in the diagram at the left below. The wheel rotates counterclockwise at 3 1/2 rev/sec. Thus, when $t = 1/21$ sec, the rod is situated as in the diagram at the right below.

(a) How far is the right end of the rod (the point B) from the center of the wheel at time $t = 1/21$ sec?

(b) Express the position of the right end of the rod as a function of t.

(c) Express the speed of the right end of the rod as a function of t.

(d) Use parts (a), (c) and the tangent line approximation to find the distance from B to the center of the wheel at time $t = 1/20$ sec.

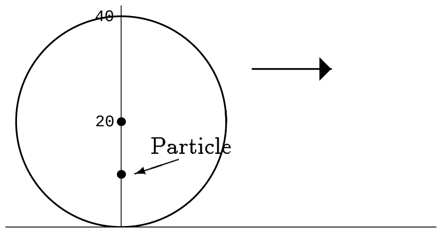


6. (20 points) A wheel 20 cm in radius rolls along a flat surface without slipping at a speed of 5 cm/sec. A particle is caught exactly half way up one of the spokes. At time $t = 0$ the particle is at its lowest point (a height of 10 cm above the surface). The drawing below shows the wheel at time $t = 0$.

(a) Find an expression for the horizontal position of the particle (its x-coordinate) at time t.

(b) What is the x-coordinate of the particle when $t = 3$ sec? In this part and the next, carry out your computations to five places beyond the decimal point.

(c) Suppose that you want to find the time when the particle has moved a horizontal distance of 10 cm, i.e., when its x-coordinate is 10. Write a function $f(t)$ that equals zero at this instant. Then use Newton's method with $t_0 = 3$ to find successive approximations t_1, t_2. Also compute the horizontal position of the particle at time t_2. Please show your work clearly.


40
20
Particle

Practice Final Exam #2.

(100 points; time = 3 hours, checking your work carefully)

1. (15 points) Evaluate: (a) $\int \frac{dt}{(1-2t)^3}$ (b) $\int_0^{\pi/6} \sqrt{\cos(3x)}\sin(3x)\,dx$.

2. (10 points) A wheel 9 inches in radius is rolling along a pavement without slipping. Its angular velocity at time t seconds is $60 + 20\sin(\pi t)$ rev/sec.

(a) How many revolutions does the wheel go through between $t = 0$ and $t = \frac{1}{3}$ sec?

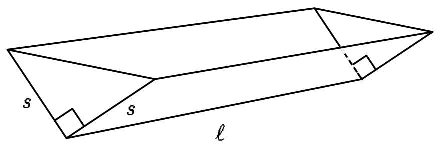
(b) How many radians does the wheel rotate during this time?

(c) How far does the wheel roll during this time? (Give your answer in feet.)

3. (10 points) A girl flies a kite at a height of 240 ft, the wind carrying the kite horizontally away from her at a rate of 20 ft/sec. How fast must she let out the string when the kite is 400 ft away from her?

4. (10 points) You are making a rectangular box with square bottom. If the bottom has side 2 ft and the height is 4 ft, then the total area of all six sides is 40 ft^2. If you decrease the height to 3.9 ft, what should the dimensions of the square bottom be in order for the surface area to remain the same? Use implicit differentiation and the tangent line approximation.

5. (10 points) You are building a trough whose sides are $45° - 45° - 90°$ triangles (see the diagram at the right). Those triangular sides are three times more expensive (in cost per unit area) than the two rectangular pieces that run the length of the trough. You want the trough to hold 6 ft^3. What dimensions s and ℓ will minimize the cost?



6. (25 points) The purpose of this problem is to sketch the function $y = f(x) = x^2 + \sin(2x)$ between $x = -\pi/2$ and $x = \pi/2$.

(a) On the same graph, sketch both the curve $y = x^2$ and the curve $y = \sin(2x)$ between $x = -\pi/2$ and $x = \pi/2$.

(b) What equation must be satisfied in order for $y = f(x) = x^2 + \sin(2x)$ to have a maximum or minimum at x?

(c) Find the x- and y-coordinates of all points of inflection of $y = f(x) = x^2 + \sin(2x)$ between $x = -\pi/2$ and $x = \pi/2$.

(d) Give a value of x near a minimum of $y = f(x) = x^2 + \sin(2x)$. Do this by finding a minimum of $\sin(2x)$. Call this value x_0.

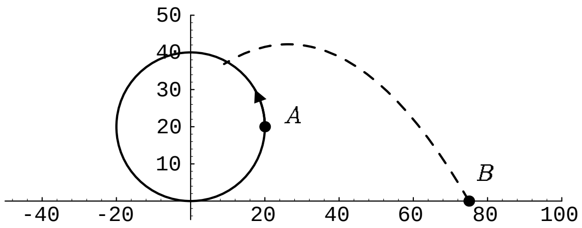
(e) Use Newton's method with two iterations to get a more accurate value x_2 for the x-coordinate of the minimum point of $y = f(x) = x^2 + \sin(2x)$.

(f) Do you think the function $y = f(x) = x^2 + \sin(2x)$ has any other maxima or minima besides the minimum you found in part (e)? (You are not expected to do any computations or take a long time on this part.)

(g) Sketch the graph of $y = f(x) = x^2 + \sin(2x)$.

(Continue to Problem 7 on the next page)

7. (20 points) You are standing at B, a distance of 75 ft from the bottom of a ferris wheel which is 20 ft in radius. Your arm is at the same level as the bottom of the ferris wheel. At time $t = 0$ your friend is at the point A rotating counterclockwise at the rate of 1 revolution every 12 seconds. At that instant you throw a ball to her. You throw the ball at 60 ft/sec at an angle of 60° above the horizontal. Take $g = 32$ ft/sec^2, and neglect air resistance.



(a) What is your friend's angular velocity in rad/sec?

(b) Using the x- and y-coordinates in the diagram, write parametric equations for your friend's position.

(c) Find the horizontal and vertical components of the velocity of the ball at time $t = 0$. Notice that the horizontal motion is in the *negative* x-direction. Round off your answers to the nearest ft/sec.

(d) Write parametric equations for the x- and y-coordinates of the position of the ball after t sec.

(e) At time $t = 2$ sec, how far is the ball from your friend?

(f) Express the distance from the ball to your friend at time t in terms of t.

(g) Suppose you want to find the closest distance the ball gets to your friend. Find a function of t that must be equal to zero at the moment of this minimum distance. This is the function that would be put into the Newton's method formula (with initial guess $t_0 = 2$ sec) to find the time when the ball is closest to your friend.

Practice Final Exam #3.

(100 points in all, time = 3 hours, checking your work carefully)

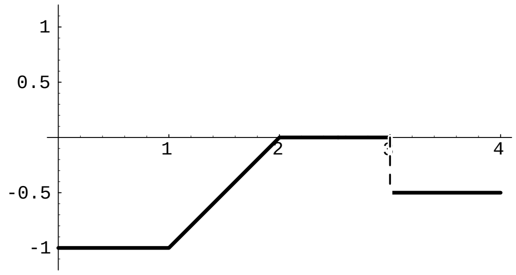
1. (15 points) Evaluate: (a) $\displaystyle\int \frac{dz}{\sqrt{1-7z}}$ (b) $\displaystyle\int_0^{\pi} \sin(x/4)\cos^2(x/4)\,dx$.

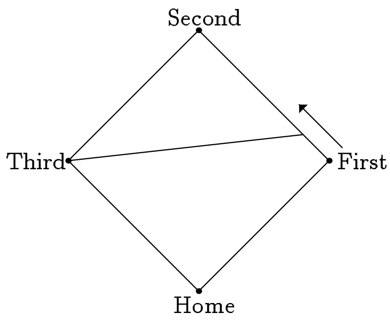
2. (15 points) At time $t = 0$ an object is moving to the right at 2 m/sec. It then decelerates according to the graph below, which shows rightward acceleration as a function of t. Here $a(t)$ is measured in m/sec^2.

(a) Draw the graph of the velocity function $v(t)$ during these 4 seconds.

(b) Find formulas for $v(t)$ for t in the intervals (0,1), (1,2), (2,3) and (3,4).

(c) How far does the object travel between $t = 0$ and $t = 4$?





3. (10 points) A baseball diamond is a square 90 ft on a side. A player runs from first base to second base at 16 ft/sec. At what rate is the player's distance from third base decreasing when the player is 20 ft from first base?

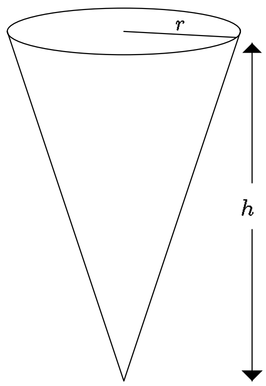
4. (15 points) Two legs of a right triangle are 7 and 24 (so the hypotenuse is 25). If you want to shorten the shorter leg to 6.980 and keep the perimeter of the right triangle (the sum of the three sides) constant, how long should you make the longer leg? Use the tangent line approximation and implicit differentiation.

5. (20 points) In this problem you want to find the best ratio of height to radius $x = h/r$ so that a cone whose side has fixed area holds maximum volume. Use the formulas $\frac{1}{3}\pi r^2 h$ for the volume of a cone and $\pi r\sqrt{r^2+h^2}$ for the side of a cone.

(a) Express both the volume and area of side in terms of r and x (without h).

(b) Suppose the side area must be a constant C. Solve for r in terms of x. (The constants π and C should also appear in your formula.)

(c) Find the ratio of height to radius for which the cone of side area C holds the most volume.



(Continue to Problems 6 and 7 on the next page)

6. (10 points) Use Newton's method with two iterations to find a number that is equal to the sum of its square root and its cube root. 3 is a good initial guess, since $\sqrt{3}+\sqrt[3]{3}=3.1743$.

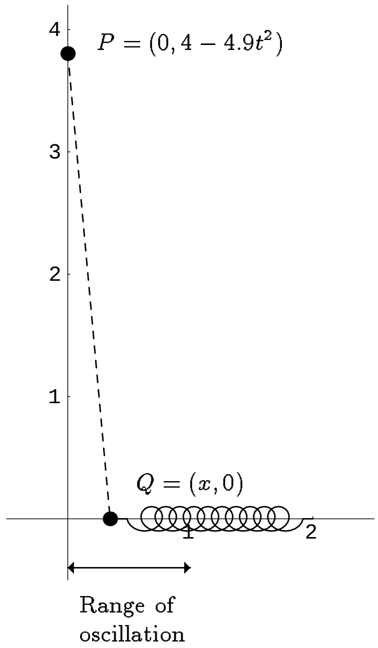
7. (15 points) At time $t=0$ an object P is dropped from a height of 4 m on the y-axis. At the same instant an object Q at the origin is released so that it can oscillate back and forth on a horizontal spring. The point Q moves sinusoidally, returning to the origin each 1 sec. When the spring is most compressed, Q is at 1 m on the x-axis. Your purpose in this problem is to find the closest distance between P and Q.

(a) Write a formula for Q's position as a function of time in the form

$$x = A\sin\left(\frac{2\pi}{B}(t-C)\right)+D\,.$$

(b) Write a formula for the **square** of the distance from P to Q as a function of time.

(c) Find a function of t that is zero at the instant when P and Q are closest. This is the function that would be put into Newton's method to find the time when P and Q are closest.



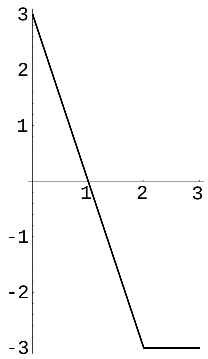
Practice Final Exam #4.

(100 points in all, time = 3 hours, checking your work carefully)

1. (20 points) Evaluate (in part (b) express your answer in terms of B):

$$\text{(a)} \int_0^3 \frac{dt}{\sqrt[4]{1+5t}} \quad \text{(b)} \int_0^{B/6} \cos\left(\frac{2\pi x}{B}\right)\sin\left(\frac{2\pi x}{B}\right)\,dx.$$

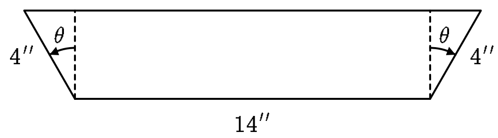
2. (15 points) At time $t = 0$ an object is moving to the right at 3 m/sec. It then accelerates according to the graph at the right, which shows rightward acceleration as a function of t (measured in m/sec^2).



(a) Find a formula for $v(t)$ when t is between 0 and 2 and when t is between 2 and 3.

(b) How far does the object travel in the 3 seconds?

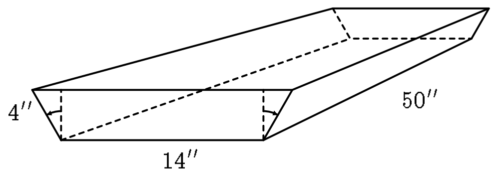
3. (15 points) The trough shown at the right is to be constructed from a piece of metal 50 inches long and 22 inches wide. The trough is to be made by turning up strips of width 4 inches to make equal angles θ with the vertical. **See the diagram at the right.**



(a) Express the volume of the trough in terms of the angle θ.

(b) Let $X = \sin\theta$. Find an expression of the form $aX^2 + bX + c$ which must equal 0 if θ is chosen so as to maximize the volume of the trough.

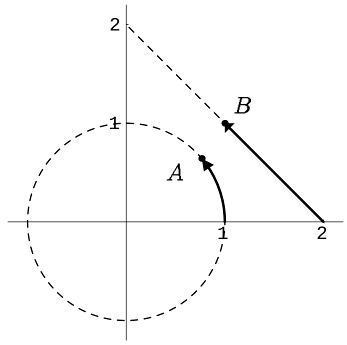
(c) Find the maximum possible volume of the trough. (In this problem you will need to use the quadratic formula $(-b \pm \sqrt{b^2 - 4ac})/2a$ to solve for $X = \sin\theta$.)



4. (10 points) Use Newton's method to find the side s of a cube for which the sum of its volume and its surface area is 1500. Use enough iterations so as to be sure that your answer is accurate to four significant figures.

(Continue to Problems 5 and 6 on the next page)

5. (15 points) The point A revolves counterclockwise around the unit circle at the rate of 1 revolution every 9 seconds. At $t = 0$ it starts at the point $(1, 0)$. The point B starts at the point (2,0) and travels in a straight line at constant velocity, reaching the point (0,2) after 2 seconds.



(a) Find parametric equations for both points A and B.

(b) Find a function which must be equal to zero at the instant when A and B are closest. This is the function that you could put into Newton's method to find the instant when A and B are closest.

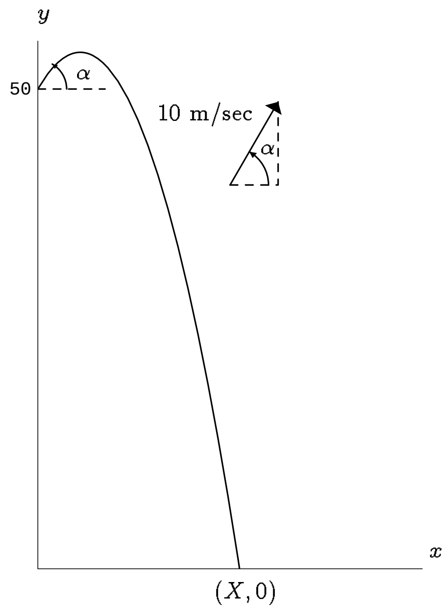
6. (25 points) At time $t = 0$ a ball is thrown out of a 50-meter high window at a velocity of 10 m/sec at an angle of α radians above the horizontal. Take the ground to be the x-axis and the side of the building from which the ball was thrown to be the y-axis. Neglect air resistance, and take $g = 9.8$ m/sec^2.

(a) Find parametric equations for the x- and y-coordinates of the ball at time t. Some trig functions of α should appear in your equations.

(b) Let T be the time when the ball hits the ground. Express T as a function of α (you will need the quadratic formula

$$(-b \pm \sqrt{b^2 - 4ac})/2a,$$

where the sign in front of the square root should be chosen so as to give positive T).



(c) Let X be the distance of the ball from the building when it hits the ground. Express X as a function of α.

(d) Suppose that α is measured to be $60°$ with an error of $\pm e$ degrees: $\alpha = 60° \pm e°$. Use the tangent line approximation to determine how sensitive X is to the error in α. In other words, find the constant of proportionality which relates the error in X (in meters) to the error e in α (in degrees).