Contents

Answers

1. Answers to Homework Problems

Answers to Selected Problems in Chapter 1. 1: m, b and distance are: (a) 1.5, -3, $\sqrt{13}$; (b) -3, 2, $\sqrt{10}$; (c) 1, 0, $2\sqrt{2}$; (d) 0, 3, 6; (e) 1, 1, $5\sqrt{2}$; (f) -3, 0.02, $0.02\sqrt{10}$. 2: y- and x-intercepts are: (a) 2, -1; (b) 6, 6; (c) 1/2, -1; (d) 3/2, none; (e) -2, -3. 3: (a) $x^2+y^2=9$, (b) $(x-5)^2+(y-6)^2=9$, (c) $(x+5)^2+(y+6)^2=9$, (d) $x^2+(y-3)^2=9$ (or $x^2+y^2-6y=0$), (e) $x^2+y^2+6y=0$, (f) $x^2-6x+y^2=0$. 4: radius and center are: (a) 5, $(0,-5)$; (b) 7, $(5,0)$; (c) 5, $(3,4)$.
5: $y=1.8x+32$. 6: $y=0.15x+10$. 7: $y=0.08x-3.2$. 8(b); $y=3x+120$. 9: (a) $y=0$ for $0\le x\le 100$, $y=0.1x-10$ for $100\le x\le 1000$, $y=x-910$ for $x\ge 1000$; (b) he's introducing a "negative income tax" as a rather cheap way of getting good PR — so that, for example, a penniless person receives 10 gold coins from the King (since the y-intercept is -10). 10:(a) $P=-0.0001x+2$; (b) $x=-10000P+20000$. 11: $y=0.15x$ for $0\le x\le 19450$, $y=0.28x-2528.5$ for $19450\le x\le 47050$, $y=0.33x-4881$ for $47050\le x\le 97620$.

Answers to Selected Problems in Chapter 2. 1: $[1.5,\infty)$. 2: $x\ne -1$. 3: $x\ne \pm 1$. 4: $(-\infty,0)$. 5: all x. 6: $[0,\infty)$. 7: $[h-r,h+r]$. 8: x negative or in $[1,\infty)$. 9: $\left(-\frac{1}{3},\frac{1}{3}\right)$. 10: x nonnegative but $x\ne 1$. 11: x nonnegative but $x\ne 1$.

Answers to Selected Problems in Chapter 3. 1: (a) -5, (b) -2.47106, (c) -2.40679, (d) -2.40068, (e) -2.4. 2: (a) $-4/3$, (b) $-24/7$, (c) $7/24$, (d) $3/4$. 3: (a) 5, (b) 4.1, (c) 4.01, (d) 4.001; limit=4. 4: (a) -10.29, (b) -9.849, (c) -9.8049; $-9.8-4.9\Delta t$; -9.8 m/sec. 5: (a) -0.107527, (b) -0.110742, (c) -0.111074; $-1/\big(3(3+\Delta x)\big)$; $f'(3)=-1/9$. 6: $(2+\Delta x)+1/(1+\Delta x)$; $f'(1)=3$. 7: (a) 3.31, (b) 3.003001, (c) 3.0000300001; $3+3\Delta x+(\Delta x)^2$; $f'(1)=3$.

Answers to Selected Problems in Chapter 4. *(first two problems)*
1: $f'(x)=-x/\sqrt{169-x^2}$. 2: (a) $f'(t)=-9.8t$, (b) $f'(x)=2x+(1/x^2)$, (c) $f'(x)=2ax+b$, (d) $f'(x)=3x^2$.

Answers to Selected Problems in Chapter 5. 1:(a) 4.4, (b) 0.9, (c) 1.01, (d) 0.0099, (e) 2.0025, (f) 5.1, (g) 4.9, (h) 2.001. 2: 3.94 3: 0.9433. 4: 15 ft 2 in. 5: 208.9 ft. 6: $25\pi\pm 4\pi(\Delta x)$. 7: ± 30, $\pm 1\%$ and $\pm 3\%$.
8: (a) 5 sec, (b) ± 0.025 sec, $\pm 0.5\%$, (c) $\pm\frac{1}{2}p\%$. 9: (a) $r=CA^{1/2}$, where $C=1/\sqrt{\pi}$, (b) $\pm 0.5\%$, (c) $\pm\frac{1}{2}p\%$. 10: 8 ± 0.15. 11: $\pm 3p\%$. 12: (a)$\pm\frac{1}{2}p\%$, (b) $\pm\frac{1}{3}p\%$. 13: (a) $\pm p\%$, (b) $\pm 3p\%$, (c) $\pm 2p\%$. 14: $\pm\frac{2}{3}p\%$. 15: (a) $\pm 2p\%$, (b) $\pm p\%$, (c) $\pm p\%$. 16: 1 83/84. 17: $\Delta x=-1/12$ cm.

Answers to Selected Problems in Chapter 6. 1: (a) $4x^3-9x^2+x+7$, (b) $3x^2-4x+2/\sqrt{x}$, (c)-(d) $6x(x^4+2x^2+1)$,
(e) $\sqrt{169-x^2}-x^2/\sqrt{169-x^2}$, (f) $(2x-4)\sqrt{25-x^2}-x(x^2-4x+5)/\sqrt{25-x^2}$,
(g) $(-x^2+2x+2)/(1-x)^2$, (h) $-25/\left(x^2\sqrt{25-x^2}\right)$, (i)-(j) $-\sqrt{x}/\sqrt{169-x^2}-\frac{1}{2}\sqrt{169-x^2}/x^{3/2}$,
(k) $\frac{1}{2}\left(3x^2-2x+(1/x^2)\right)/\sqrt{x^3-x^2-(1/x)}$, (l) $300x/(100-x^2)^{5/2}$,
(m) $(\frac{1}{3}+x^2)(x+x^3)^{-2/3}$. (n) $\left((x^2+1)^2+\sqrt{1+(x^2+1)^2}\right)^{-1/2}x(x^2+1)\left(2+\left(1+(x^2+1)^2\right)^{-1/2}\right)$.

2: (a) $-9.8t+20$, -9.8; (b) a, 0; (c) $\frac{1}{2}a(at+b)^{-1/2}, -\frac{1}{4}a^2(at+b)^{-3/2}$; (d) $\frac{1}{2}(2at+b)(at^2+bt+c)^{-1/2}$, $a(at^2+bt+c)^{-1/2} - \frac{1}{4}(2at+b)^2(at^2+bt+c)^{-3/2}$, which simplifies to $\frac{1}{4}(4ac-b^2)(at^2+bt+c)^{-3/2}$.
3: $v(t) = -5+2t$, $a(t) = 2$; the bicycle is moving leftward initially at 5 m/sec, but its speed is decreasing; after 2.5 sec it has slowed to 0 m/sec, at which point it reverses direction and starts moving to the right.
4: $2 + \frac{\pi}{4}\Delta x$. 5: $\pm 0.96\Delta x$. 6: $\frac{500}{3}\pi \pm 40\pi(\Delta x)$. 7: (a) $f(t) = \sqrt{100+(78-4.9t^2)^2}$; (b) $-9.8t(78-4.9t^2)/\sqrt{100+(78-4.9t^2)^2}$.

Answers to Selected Problems in Chapter 7. 1: (a) 1.992, (b) 5.0129, (c) 4.00133. 2: (a) $y' = x/y$, (b) $y' = -(2x+y)/(x+2y)$, (c) $y' = (3x^2+y^2-2xy)/(3y^2+x^2-2xy)$. 3: $y' = 1$. 4: $y = -\frac{1}{13}x + \frac{14}{13}$, 0.9923. 5: B is 4 times as strong as A. 6: $y' = 3/19$. 7: (a) $dT/dh = -38.64°$F/hour, (b) $h = -0.0259T+5.493$. 8: (a) $\sqrt{100+(10+0.3t)^2}+\sqrt{(x-10)^2+(10-0.3t)^2}$ =const, (b) 12.492 sec.

Answers to Selected Problems in Chapter 8. 1: (a) $\pi/12$, (b) $2\pi/3$, (c) $\pi/2$, (d) 20π. 2: (a) 25 rad/sec (b) 6.944 rad/sec. 3: (a) $30\sin\theta$, (b) $\tan\theta$, (c) $60(1-\cos\theta)$. 4: $\sin(\alpha+\beta) = \overline{BC} = \overline{DF} = \overline{DE} + \overline{EF} = \overline{AE}\sin\alpha + \overline{BE}\cos\alpha = \cos\beta\sin\alpha + \sin\beta\cos\alpha$. 6: warmest: 62° on July 12; coolest: −12° on January 10. 7: $F = 50\sin\left(\frac{\pi}{2}t\right) + 150$, $R = 2500\sin\left(\frac{\pi}{2}(t-3)\right) + 7500$. 8: (a) $20\sin\left(\frac{2\pi}{360}(t-109)\right)+40$; (b) $2\sin\left(\frac{2\pi}{360}(t-81)\right)+7$; (c) $5\sin\left(\frac{2\pi}{24}(t-11)\right)+20$; (d) $10\sin(5\pi t)$; (e) $0.4\sin\left(25(t-\frac{\pi}{50})\right)+0.4$ (or, equivalently, $0.4(1-\cos(25t))$).

Answers to Selected Problems in Chapter 9. 1: 0.96509 (exact value=0.96569). 2: (a) $-\csc^2 x$, (b) $-\cot x\csc x$. 3: (a) 0.46977, (b) 0.484885, (c) 0.93018. 4: (a) $\frac{\pi}{9}\cos\left(\frac{2\pi}{360}(t-109)\right)$; (b) $\frac{\pi}{90}\cos\left(\frac{2\pi}{360}(t-81)\right)$; (c) $\frac{5\pi}{12}\cos\left(\frac{2\pi}{24}(t-11)\right)$; (d) $50\pi\cos(5\pi t)$; (e) $10\sin(25t)$.
5: (a) $\cos^2 x - \sin^2 x$, (b) $-\sin(x)\cos(\cos x)$, (c) $\left(x\sec^2 x + \tan x\right) / \left(2\sqrt{x\tan x}\right)$,
(d) $\Big((1+\sin x)\sec^2 x - \sin x\Big)/(1+\sin x)^2$. 6: 0.7632. 7: 1.241. 8: (b) $0.5 = 2\sin(\pi t)\sin(\pi x/3)$; (c) $x = 0.5$; (d) $dx/dt = -3$; (e) $x = 0.5 + (0.2-\frac{1}{6})(-3) = 0.4$.

Answers to Selected Problems in Chapter 10. 1: (a) $x = 2+3t$, $y = 3-2t$; (b) $x = 1+6t$, $y = 2+2.5t$; (c) $x = 5\sqrt{3}t$, $y = 5t$. 2: (a) 25 rad/sec; (b) $\frac{25}{2\pi}$ rev/sec; (c) 4π m/sec; (d) 21.6 km/hr. 3: (a) 34.3 mph; (b) 55 rad/sec, 8.75 rev/sec. 4: (a) $20t$; (b) $15\pi t$; (c) $7.5t^2$. 5: (a) $x = 3\cos 0.5t$, $y = 3\sin 0.5t$; (b) $x = 3\sin 4\pi t$, $y = 3\cos 4\pi t$; (c) $x = -3\sin 0.4\pi t$, $y = -3\cos 0.4\pi t$; (d) $x = 3\sin 20\pi t$, $y = -3\cos 20\pi t$. 6: (a) $x = 10t$, $y = -4.9t^2 + 24.5t + 1$; (b) 2.5 sec; (c) 5.04 sec. 7: (a) $x = 49.5t$, $y = -16t^2 - 49.5t + 163$; (b) $(-32t-49.5)/49.5$; (c) 52.9°, 58.7°; (d) 124 ft/sec at 66.4° below horizontal (hits ground at $t = 2$). 8: (a) $x = (v\cos\alpha)t$, $y = \frac{1}{2}gt^2 + (v\sin\alpha)t$; (b) $2v\sin\alpha/(-g)$; (c) $2v^2\sin\alpha\,\cos\alpha/(-g)$.

Answers to Selected Problems in Chapter 11. 1: (b) $\sin\theta/(1-\cos\theta)$. 2: (a) $x = 2t$; (b) $y = 3\sin(4\pi(t-\frac{3}{8}))$; (c) $y = 3\sin(2\pi(x-\frac{3}{4}))$. 3: $y = 3\sin(4\pi(\sqrt{x}-\frac{3}{8}))$. 4: $x = 2\sin(\pi t)$, $y = 100 - 16t^2 + 2\cos(\pi t)$. 5: $x = 6t - 0.5\sin(8t)$, $y = 0.75 - 0.5\cos(8t)$. 6: $x = 10t + 0.25\sin(40t)$, $y = 0.25 - 4.9t^2 + 0.25\cos(40t)$.

Answers to Selected Problems in Chapter 12. 1: 0.0199 cm/sec. 2: 0.25 m/sec. 3: 80π mi/min (=4.2 mi/sec). 4: 6.71 ft/sec. 5: 2.12 cm/sec. 6: 0.65 ft/sec 7: 7.91 m/sec 8: 1.172 m/min 9: 0.5454 m/min 10: 6 ft/sec, 2.5 ft/sec 11: 69.4 mph 12: 0.1234 ft³/sec. 13: (a) $(a\dot{a} + b\dot{b} + 0.5a\dot{b} + 0.5b\dot{a})/\sqrt{a^2+b^2+ab}$; (b) $(a\dot{a} + b\dot{b} - a\dot{b}\cos\theta - b\dot{a}\cos\theta + ab\dot{\theta}\sin\theta)/\sqrt{a^2+b^2-2ab\cos\theta}$. 14: 81.6 m/sec 15: (a) $y = a\cos(\theta) - a\sin(\theta)\cot(\beta+\theta)$ (or else, using the law of sines, an equivalent answer is: $y = a\sin\beta/\sin(\theta+\beta)$); (b) $dy/dt = \Big(-a\sin(\theta)(1-\csc^2(\beta+\theta)) - a\cos(\theta)\cot(\beta+\theta)\Big)\,d\theta/dt$ (or else: $dy/dt = -a\Big(\sin\beta\cos(\theta+\beta)/\sin^2(\theta+\beta)\Big)\,d\theta/dt$); (c) 3.79 cm/sec. 16: (a) $a(d\theta/dt)$; (b) closing at a little over 15 rad/sec; (c) no violation of relativity, because no part of the scissors is moving faster than 300 km/sec. 17: (a) -50 m/sec; (b) 68 m/sec; (c)... 18: (a) $(a\dot{a}+b\dot{b})/\sqrt{a^2+b^2+h^2}$; (b) $(a\dot{a}+b\dot{b}+h\dot{h})/\sqrt{a^2+b^2+h^2}$.

Answers to Selected Problems in Chapter 13. 1: Both $f'(x) = 0$ and $f''(x) = 0$ when $x = 0$. If n is even, the origin is a minimum, and the curve is always concave up. If n is odd, the origin is a point of inflection, and the curve is concave up for $x > 0$ and concave down for $x < 0$.
2: (a) min at $(0.5, -0.25)$, always concave up, no inflection;

(b) max at (1,4), min at $(-1, 0)$, inflection at $(0, 2)$, conc up for $x < 0$, conc down for $x > 0$;

(c) max at (2,20), min at (4,16), inflection at (3,18), conc up for $x > 3$, conc down for $x < 3$;

(d) max at (0,3), min at $(\pm 1, 2)$, inflections at $(\pm 0.577, 2.444)$, conc down for $-0.577 < x < 0.577$, otherwise conc up, symmetric with respect to y-axis;

(e) min at $(1, -1)$, inflections at (0,0) and $(2/3, -0.5926)$, conc down for $0 < x < 2/3$, otherwise conc up;

(f) no max, min or inflections, y-axis is asymptote, conc up for $x < 0$, conc down for $x > 0$, the line $y = x$ is a slanted asymptote;

(g) no max or min, inflections at $(\pm 1, 2)$, y-axis is asymptote, conc up for $x < -1$ and $x > 1$, otherwise conc down, symmetric with respect to y-axis;

(h) min when $x = 7\pi/12$ (and $x = 7\pi/12\pm$any multiple of π), max when $x = 11\pi/12$ (again can add or subtract any multiple of π), inflections when $x = \pi/4$, $3\pi/4$ ($\pm$ any multiple of π), conc up for $\pi/4 < x < 3\pi/4$, general appearance is oscillation around the slanted line $y = -x$;

(i) max at all points $(\frac{\pi}{4} \pm 2k\pi, \sqrt{2})$ (where k is any integer), min at $(\frac{5\pi}{4} \pm 2k\pi, -\sqrt{2})$, inflections at $(\frac{3\pi}{4} \pm k\pi, 0)$, you get a sinusoidal function;

(j) asymptotes at odd multiples of π, max when $x = -\frac{\pi}{2} \pm 2k\pi$, min when $x = \frac{\pi}{2} \pm 2k\pi$, inflections at even multiples of π, in each section between asymptotes the left half is conc down, the right half is conc up.
5: (a) max at (0,0), min at $(\pm\sqrt{10/3}, -500/27)$; (b) $x > \sqrt{2}$ and $x < -\sqrt{2}$; (c) $(\pm\sqrt{2}, -12)$; (d) use 2nd derivative test.

Answers to Selected Problems in Chapter 14. 1: (a) max value $f(2) = 5$; (b) min value $f(0) = 1$. 2: $P/4$ by $P/4$. 3: $\sqrt[3]{2V}$ by $\sqrt[3]{2V}$ by $\frac{1}{2}\sqrt[3]{2V}$; ratio=1/2. 4: $l^2/8$. 5: $x = 2000$, $P = \$6$, profit=\$5000. 6: $r\sqrt{2}$ by $r\sqrt{2}/2$, area=r^2. 7: $h/r = 2$. 8: $h/r = 8/\pi$. 9: 4/27. 10: Take $x = a$, where $f(x)$ has a corner (if $w > v$, then $f'(x)$ is never zero). 11: $h/r = 2$; $h/r = 3.5$. 12: (a) $a/6$ on a side; (b) $\left(a + b - \sqrt{a^2 + b^2 - ab}\right)/6$ on a side. 13: $0.5 - \frac{\pi}{4}k$ provided that $k < 2/\pi$; otherwise, the optimal window consists of only the semicircular piece. 14: $\theta = 30°$, max area= $l^2\sqrt{3}/12 \approx 0.144\, l^2$, bigger than in Problem 3. 15: 0.00139 hours ($\approx$ 5 sec), misses by 0.5726 mi. 16: ratio=a/b. 17: $\frac{2}{3}r\sqrt{3}$ by $\frac{2}{3}r\sqrt{6}$. 18: $1/\sqrt{3}$ ($\approx$58%). 19: 18 in $\times$ 18 in $\times$ 36 in. 20: $r = 2.71$, $h = 10.84$. 21: $h/r = \sqrt{2}$. 22: $h = 1.1798R$, $r = 0.9837R$, 28.54% 23: $c/(1+\sqrt[3]{b/a})$ from A. 24: 1/2. 25: $x = 2500$, $P = \$5$, profit=\$7000. 26: Snell's law: $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$.

Answers to Selected Problems in Chapter 15. 1: $x_1 = 1.4765432$, $x_2 = 1.4757740$, $x_3 = 1.4757732$. 2: $x_1 = 2.25$ (exactly), $x_3 = x_2 = 2.19$ to three significant figures ($x_2 = 2.19369\ldots$, $x_3 = 2.19192\ldots$). 3: $\theta_1 = 1.9359512$, $\theta_2 = 1.9345639$, $\theta_3 = 1.9345632$. 4: Hint: Set the slope of the line joining $(-\pi/2, -1)$ to $(x, \sin x)$ equal to the derivative of $\sin x$, then multiply through by the denominator to get the $f(x)$ in Newton's method: $f(x) = (\frac{\pi}{2} + x)\cos x - \sin x - 1$; then $x_0 = 0.7$, $x_1 = 0.7633$, $x_3 = x_2 = 0.7603$. 5: $t = 0.5062$ sec, height=6.9938 ft. 6: take $t_0 = 0.7$, then $t_1 = 0.68674, t_2 = 0.68726$. 7: $x = 40\sin(\pi t/10) - 10\sin(\pi t/3)$, $y = 50 - 40\cos(\pi t/10) - 10\cos(\pi t/3)$, 9.2655 sec, 98.56 ft. 8: $l = \left(9/\sin(\frac{\pi}{3} - \theta)\right) + \left(7/\sin\theta\right)$, 0.49254 rad ($= 28.2°$), 31.89 ft

Answers to Selected Problems in Chapter 16. 1: (a) $-\frac{1}{3x^3} + \frac{2}{5}x^{5/2} + 5x + C$, (b) $-0.1(1-t)^{10} + C$, (c) $\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$, (d) $-0.3(1 - 5t)^{2/3} + C$, (e) $\frac{1}{4}\sin^4 x + C$, (f) $-\frac{1}{3}(100 - x^2)^{3/2} + C$, (g) $-\frac{2}{3}\sqrt{1 - x^3} + C$, (h) $-\frac{B}{2\pi}\cos\left(\frac{2\pi}{B}(t - C)\right) + Dt + C'$, (i) $-\frac{20}{3}\left(1 - \sin\frac{t}{10}\right)^{3/2} + C$, (j) $\frac{1}{\pi}\sin\big(\sin(\pi t)\big) + C$. 2: $-\frac{1}{\pi}\cot(\pi t) + C$. 3: $-\frac{1}{2}g(t - t_0)^2 + v_0(t - t_0) + s_0$. 4: $\frac{1}{3}t - \frac{1}{2} + 1/(6t^2)$. 5: (a) $\frac{1}{6}kt^3 + v_0 t + s_0$, (b) $\frac{1}{15}(2t+1)^{5/2} + (v_0 - \frac{1}{3})t + s_0 - \frac{1}{15}$, (c) $-\frac{1}{25}\sin(5t) + (v_0 + \frac{1}{5})t + s_0$, (d) $50\tan(0.1t) + (v_0 - 5)t + s_0$.

Answers to Selected Problems in Chapter 17. 1: (a) 6.25 vs. 16/3=5.333, (b) 5.8125 vs. 5.333, (c) 4.146 vs. 5.333, (d) 1.239 vs. 1, (e) 1.125 vs. 1. 2: (a) 55, (b) 25/12, (c) $\frac{3}{2} + \frac{\sqrt{3}}{2}$. 3: (a) 28.5, (b) 14/9, (c) 2/3, (d) 0.24, (e) $2/\pi$. 4: (a) 2/3, (b) $2 - \sqrt{2}$, (c) $\frac{4}{\pi} + \frac{4}{3}$, (d) $\pi - 2$. 5: (a) 13/3, (b) 56/15, (c) 0, (d) $1/4\pi$, (e) 3/4, (f) 9. 6: (a) 14600 degree-days, (b) 4523 degree-days, (c) 277 degree-days. 7: (a) $a(t) = \begin{cases} 2t, & 0 \le t \le 2 \\ 4, & 2 \le t \le 4 \\ 8-t, & 4 \le t \le 8 \end{cases}$; $v(t) = \begin{cases} t^2, & 0 \le t \le 2 \\ 4t-4, & 2 \le t \le 4 \\ 8t - \frac{1}{2}t^2 - 12, & 4 \le t \le 8 \end{cases}$; $s(8) - s(0) = 88$. (b) $a(t) = \begin{cases} \sin t, & 0 \le t \le 0.5110 \\ 1-t, & 0.5110 \le t \le 1 \end{cases}$; $v(t) = \begin{cases} 1 - \cos t, & 0 \le t \le 0.5110 \\ t - \frac{1}{2}t^2 - 0.2527, & 0.5110 \le t \le 1 \end{cases}$; $s(1) - s(0) = 0.1234$.

Answers to Selected Problems in Chapter 18. 1: (a) $\sin^2 u$, (b) $\sqrt{1+u+u^2}$, (c) $-\sqrt{1+\sin^2 u}$, (d) $2\sin^2 u$; (e) 0. 2: (a) $b/(a+bt)$, (b) $(-gt+v_0)\sin^2\left(-\frac{1}{2}gt^2 + v_0 t + s_0\right)$, (c) $3t^2\cos(t^6) - 2t\cos(t^4)$, (d) $A\alpha\cos(\alpha(t-C))\sqrt{1 + \left(A\sin(\alpha(t-C))\right)^2} - b\sqrt{1+(a+bt)^2}$. 3: (a) $\int_0^{2t}\left(20 + 10\cos^2(0.1\pi x)\right)dx$, (b) $2\left(20 + 10\cos^2(0.2\pi t)\right)$. 4: (a) $\int_0^{2t} \frac{40\,dx}{1+(0.01x-1)^2}$, (b) $\frac{80}{1+(0.02t-1)^2}$. 5: (a) $2\int_{48-16t^2}^{48} 48\cos\frac{\pi y}{96}\,dy$, (b) $3072t\cos\left(\frac{\pi}{6}(3-t^2)\right)$. 6: (a) $2\int_{-1}^{u}\sqrt{1-x^2}\,dx$, (b) $u(t) = \sin\left(\pi(t-\frac{1}{2})\right)$, (c) $2\pi\cos\left(\pi(t-\frac{1}{2})\right)\sqrt{1-\sin^2\left(\pi(t-\frac{1}{2})\right)}$.

2. Answers to Practice Midterms

Answers to Practice First Midterm #1

1. (a) $0 \le x \le 2$; (b) $-2 \le x \le 0$; (c) all x except 2 and 4; (d) $-1 < x < 1$.
2. (a) ABIJ; (b) DEFG; (c) GHI; (d) FJ; (e) J; (f) F.
3. $t = f(s) = \frac{130}{4s} - \frac{1}{2}s$, $f(5\frac{1}{12}) \approx f(5) + \frac{1}{12}f'(5) = 4 + \frac{1}{12}(-1.8) = 3.85$ ft. 4. $9\frac{7}{81} = 9.0864$ cm.

Answers to Practice First Midterm #2

1. (a) like the x^2-parabola, but shifted to have its vertex at $(-2,-4)$; (b) bottom half of ellipse, passes through ± 1 on the x-axis and -5 on the y-axis. 2. $-\pi/2 < x < \pi/2$.
3. (a) CDJ; (b) AFGH; (c) ABHI; (d) CGJ; (e) C; (f) G. 4. (a) $v_0 = 9.5$ m/sec; (b) $v_0 = f(t) = 4.9t - \frac{75}{t}$, $f(5 \pm 0.1) \approx 9.5 \pm 0.1 \cdot 7.9 = 9.5 \pm 0.79$ m/sec.
5. $y = f(x) = 1000/x$, $f(10.77) \approx f(10) + (0.77)f'(10) = 100 + (0.77)(-10) = 92.3$ m^2.

Answers to Practice First Midterm #3

1. $0.5 < x \le 2$. 2. (a) AFG; (b) CDI; (c) DE; (d) CFI; (e) C.
3. $g = f(t) = \frac{22.5}{t^2}$; (a) $f(3) = 2.50$; (b) $f(3.1) \approx f(3) + 0.1 \cdot f'(3) = 2.33$, so g is between 2.33 and 2.50 m/sec^2. 4. $y = f(x) = \sqrt{36^2 - x^2}$, $f(21.6 \pm 0.2) \approx 28.8$ in $\pm$ 0.15 in.

Answers to Practice First Midterm #4

1. (a) $-35 < x < 15$; (b) $-7 \le x \le 7$. 2. (b) when $t = 12$ (parachute opens); when $t = 0$ (drop from plane); right after parachute opens. 3. 12.8. 4. (a) $A = \frac{\pi}{4}r^2 + rs$; (b) $100\pi + 600$; (c) $s = f(r) = \frac{100\pi + 600}{r} - \frac{\pi}{4}r$, $f(21) \approx f(20) + 1 \cdot f'(20) = 26.93$ cm.

Answers to Practice Second Midterm #1

1. 19.861. 2. (a) $(-1,-1.5)$ is max; (b) $x < -1$ and $x > 0$; (c) y-axis; (d) none; (e) see bottom of page. 3. (b) $y = 20\sin\left(\frac{2\pi}{12}(t-9.5)\right) + 55$; (c) mid-March to mid-November. 4. b, $C, \cos C$ are all constants, a and c are variables, take d/dt of everything, you know $da/dt = -5.70$, you get $dc/dt = -5.55$ m/sec.

Answers to Practice Second Midterm #2

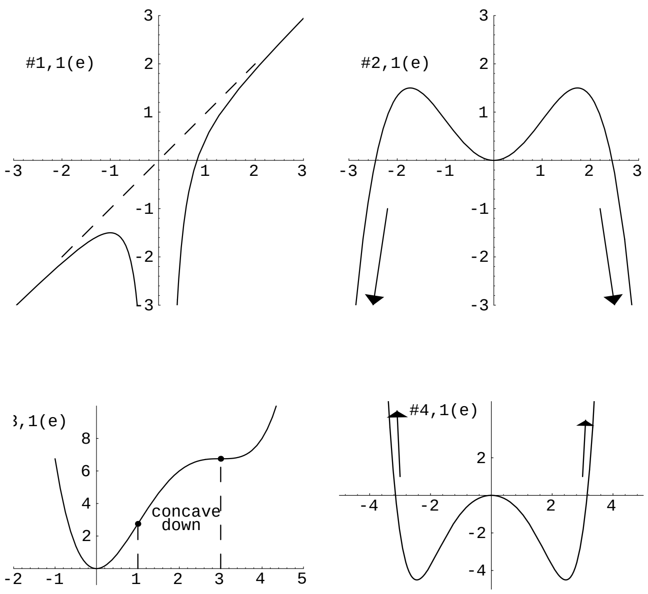
1. (a) $(0,0)$, $(\sqrt{3}, 1.5)$, $(-\sqrt{3}, 1.5)$; (b) $-\infty < x < -\sqrt{3}$, and $0 < x < \sqrt{3}$; (c) $(1, 5/6)$, $(-1, 5/6)$; (d) $-1 < x < 1$; (e) see below.
2. (a) 5π rad/sec; (b) $\dot{x} = 5(1-\cos\theta)$, $\dot{y} = 5\sin\theta$; (c) $\dot{x} = 2.50$ m/sec, $\dot{y} = -4.33$ m/sec, $dy/dx = -1.73$. 3. (a) $x = \sqrt{100 - 4y^2}$; (b) $A = \pi(100 - 4y^2)$; (c) 0.63 m^2/sec.
4. (a) $\sqrt{(x+5)^2 + y^2} + \sqrt{x^2 + (y+5)^2} = 20$; (b) $y' = 1/5$.

Answers to Practice Second Midterm #3

1. (a) $(0,0)$; (b) $(1,2.75)$ and $(3,6.75)$; (c) see bottom of page; (d) $1 < x < 3$.
2. $C = f(c)$ (a, b are const), $\pm 0.6175°$. 3. $y = 5\cot\theta$, 0.0436 ft/min.
4. (a) $y = 19.6 - 4.9t^2$; (b) $x = 2\sin(\pi(t - \frac{1}{2})) + 2$; (c) $\dot{x} = -4.443$ m/sec, $\dot{y} = -12.25$ m/sec, $dy/dx = 2.757$.

Answers to Practice Second Midterm #4

1. (a) max: $(0,0)$, min: $(\pm\sqrt{6},-4.5)$; (b) $-\sqrt{6}<x<0$ and $\sqrt{6}<x<\infty$; (c) $(\pm\sqrt{3},-2.925)$; (d) $-\sqrt{3}<x<\sqrt{3}$; (e) see bottom of page. 2. $\pi R^2 T = 1$ m^3; take d/dt of both sides; 0.804 m/hr. 3. $B=f(A)$ (a,b are const), 61.732°. 4. (a) $V=\frac{1000\pi}{3}\tan^2(\frac{1}{2}\theta)$; (b) 349 ft^3 $\pm$ 14 ft^3.



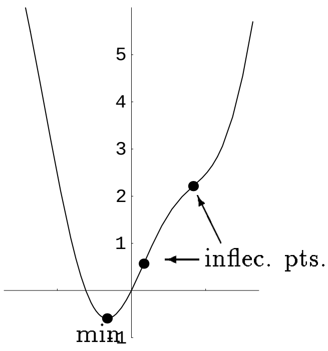
3. Answers to Practice Final Exams

Answers to Practice Final Exam #1

1: (a) -15/4; (b) $\frac{5}{\pi}\sin^2\left(\frac{\pi}{10}(t-1)\right)+C$, or else $-\frac{5}{\pi}\cos^2\left(\frac{\pi}{10}(t-1)\right)+C$. 2: -78.125 m/sec.
3: (b) $v=\frac{1}{2}t^2,\ 0\le t\le 1;\ v=t-\frac{1}{2},\ 1\le t\le 2;\ v=3/2,\ 2\le t\le 3$; (c) 8/3 m.
4: area of hexagon $=2\sqrt{3}r^2$, optimal ratio h/r is $4\sqrt{3}/\pi=2.2053\cdots$.
5: (a) 4.60555 cm; (b) $x=2\cos(7\pi t)+\sqrt{16-4\sin^2(7\pi t)}$;
(c) $dx/dt=-14\pi\sin(7\pi t)-28\pi\sin(7\pi t)\cos(7\pi t)\Big(16-4\sin^2(7\pi t)\Big)^{-1/2}$;
(d) $f'(1/21)=-48.654$, $f(1/20)\approx 4.4897$ cm.
6: (a) $x=5t-10\sin(t/4)$; (b) 8.18361 cm; (c) $t_1=3.57285$ sec, $t_2=3.55148$ sec, $x(t_2)=10.00011$ cm.

Answers to Practice Final Exam #2

1: (a) $\frac{1}{4}(1-2t)^{-2}+C$; (b) 2/9. 2: (a) $20+\frac{10}{\pi}$ rev; (b) $40\pi+20$ rad; (c) 109.25 ft. 3. 16 ft/sec. 4: 2.033 ft. 5: $s=1.59$ ft, $l=4.76$ ft.
6: (b) $2x+2\cos(2x)=0$; (c) $(\pi/12,\pi^2/144+1/2)$ and $(5\pi/12,25\pi^2/144+1/2)$; (d) $-\pi/4$; (e) $x_1=-0.5236$, $x_2=-0.5150$; (f) no other max/min; (g) graph at right:
7: (a) $\pi/6$ rad/sec; (b) $x=20\cos(\pi t/6)$, $y=20+20\sin(\pi t/6)$;
(c) $v_{0,\text{horiz}}=-30$ ft/sec, $v_{0,\text{vert}}=52$ ft/sec;
(d) $x=75-30t$, $y=-16t^2+52t$; (e) 5.6737 ft;
(f) dist$=\sqrt{(75-30t-20\cos(\pi t/6))^2+(-16t^2+52t-20-20\sin(\pi t/6))^2}$
(g) let $f(t)=\frac{d}{dt}(\text{dist}^2)=2(75-30t-20\cos(\pi t/6))(-30+(20\pi/6)\sin(\pi t/6))+$
$+2(-16t^2+52t-20-20\sin(\pi t/6))(-32t+52-(20\pi/6)\cos(\pi t/6))$.



Answers to Practice Final Exam #3

1: (a) $-\frac{2}{7}\sqrt{1-7z}+C$; (b) $(4-\sqrt{2})/3 = 0.8619$.
2: (b) $2-t$ for $0 \le t \le 1$, $0.5+0.5(2-t)^2$ for $1 \le t \le 2$, 0.5 for $2 \le t \le 3$, $2-0.5t$ for $3 \le t \le 4$; (c) 2.92 m. 3: -9.823 ft/sec. 4: differentiate $x+y+\sqrt{x^2+y^2}$ =const, get $y' = -32/49$, answer is 24.013.
5: (a) vol= $\frac{1}{3}\pi r^3 x$, area= $\pi r^2\sqrt{1+x^2}$; (b) $r = \sqrt{C/\pi}\left(1+x^2\right)^{-1/4}$. (c) $x = \sqrt{2}$.
6: Choose $f(x) = x^{1/2}+x^{1/3}-x$ in Newton's method, so that $f'(x) = \frac{1}{2}x^{-1/2}+\frac{1}{3}x^{-2/3}-1$; then $x_0 = 3$, $x_1 = 3.3163$, $x_2 = 3.3094$.
7: (a) $x = \frac{1}{2}\sin(2\pi(t-\frac{1}{4}))+\frac{1}{2}$; (b) $(4-4.9t^2)^2+(\frac{1}{2}\sin(2\pi(t-\frac{1}{4}))+\frac{1}{2})^2$; (c) $f(t) = -19.6t(4-4.9t^2)+\big(\sin(2\pi(t-\frac{1}{4}))+1\big)\pi\cos(2\pi(t-\frac{1}{4}))$.

Answers to Practice Final Exam #4

1: (a) 28/15; (b) $3B/16\pi$. 2: (a) $3+3t-\frac{3}{2}t^2$, $0 \le t \le 2$; $9-3t$, $2 \le t \le 3$; (b) 9.5 m.
3: (a) Vol= $50 \cdot 4\cos\theta(14+4\sin\theta)$; (b) $\mathrm{d(Vol)}/d\theta = -400(4X^2+7X-2)$ (where $X = \sin\theta$); (c) 2904 in^3.
4: A good initial guess is $s_0 = 10$ (since vol=1000, surface area=600). Choose $f(s) = s^3+6s^2-1500$; then $s_1 = 9.762$, $s_3 = s_2 = 9.757$.
5: (a) $x_A = \cos(2\pi t/9)$, $y_A = \sin(2\pi t/9)$, $x_B = 2-t$, $y_B = t$;
(b) $2\big(2-t-\cos(2\pi t/9)\big)\big(-1+(2\pi/9)\sin(2\pi t/9)\big)+2\big(t-\sin(2\pi t/9)\big)\big(1-(2\pi/9)\cos(2\pi t/9)\big)$.
6: (a) $x = (10\cos\alpha)t$, $y = -4.9t^2+(10\sin\alpha)t+50$; (b) $T = 1.0204\big(\sin\alpha+\sqrt{\sin^2\alpha+9.8}\big)$; (c) $X = (10\cos\alpha)T$ (write T in terms of α using (b)); (d) find $\frac{\pi}{180}\cdot\frac{dX}{d\alpha}$ and then take $\alpha = 60°$: -0.5781 (the minus sign means that a positive error in α leads to a negative error in X).