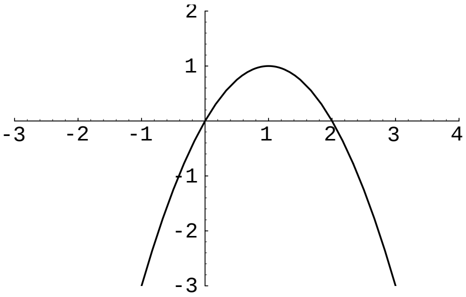
Contents

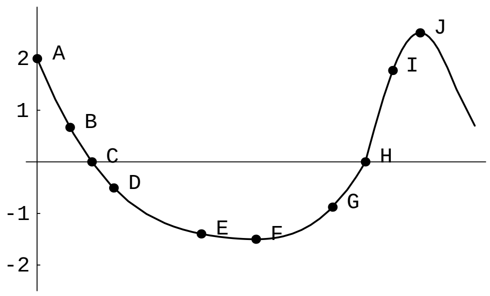
Practice First Midterms

Practice First Midterm #1 (50 points in all, time = 1 hour)



1. (12 points) The graph of $y = f(x) = 2x - x^2$ is given at the right. Find the domain of each of the following functions. You do **not** need to use any algebra in this problem.
   (a) $\sqrt{f(x)} = \sqrt{2x - x^2}$
   (b) $\sqrt{f(-x)} = \sqrt{-2x - x^2}$ (c) $\frac{1}{f(x-2)}$
   (d) $\frac{1}{\sqrt{f(x+1)}}$

2. (12 points) At time $t = 0$ an object starts from 0 moving to the right at 2 m/sec. Its velocity changes, as shown in the graph at the right. This graph shows the velocity with which the object is moving to the right as a function of time. Give the letters of **ALL** labeled points where
   (a) the object is actually moving rightward;
   (b) the object is actually moving leftward;
   (c) the object has positive (rightward) acceleration;
   (d) the object has zero acceleration;
   (e) the object's speed heading to the right is maximum;
   (f) the object's speed heading to the left is maximum.



3. (13 points) A box with square top and bottom and rectangular sides has surface made up of two $s \times s$ sides and four $s \times t$ sides. Thus, if your box has dimensions 5 ft $\times$ 5 ft $\times$ 4 ft, then the total surface area A is 130 sq ft. If you want to increase s by 1 inch to 5 ft 1 in, how should t change so that the surface area stays the same? Use the tangent line approximation, and give your answer in feet.

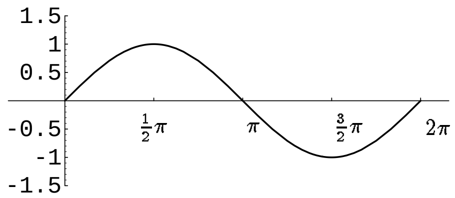
4. (13 points) A cubic container with side 9 cm holds $9^3 = 729$ cubic cm. Suppose you have a bottle containing 750 ml of liquor. (A milliliter (ml) is the same thing as a cubic cm.) Use the tangent line approximation to find the side of a cubic container needed to hold the 750 ml of liquor.

Practice First Midterm #2 (50 points in all, time = 1 hour)

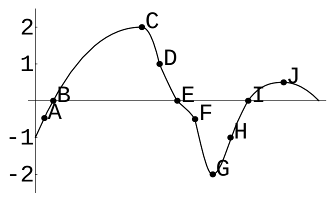
1. (8 points) Sketch the graph of the functions

$$\text{(a) } y=(x+2)^2-4, \qquad \text{(b) } y=-\sqrt{25-(5x)^2}$$

2. (6 points) The graph of the function $g(x)$ on the interval $[0, 2\pi]$ is pictured below to the left. Find the domain of $y = f(x) = 1/\sqrt{g(x+(\pi/2))}$. (This function $g(x)$ is actually the sine function restricted to the interval $[0, 2\pi]$.)



Graph for Problem 2



Graph for Problem 3

3. (12 points) At time $t = 0$ an object starts from 0 moving to the left at 1 m/sec. Its velocity changes, as shown in the graph above at the right. This graph shows the velocity with which the object is moving to the right as a function of time. Give the letters of **ALL** labeled points where

(a) the object is actually moving rightward;
(b) the object is actually moving leftward;
(c) the object has positive (rightward) acceleration;
(d) the object has zero acceleration;
(e) the object's speed heading to the right is maximum;
(f) the object's speed heading to the left is maximum.

4. (12 points) From a point **exactly** 75 m above the ground you throw a stone upward at v_0 m/sec, and you time how long it takes to hit the ground. If we neglect air resistance, the height of the stone at time t is given by the falling body formula $s = -\frac{1}{2}gt^2 + v_0t + s_0$, where $g = 9.8$ m/sec^2.

(a) If you time 5 sec for the stone to reach the ground, at what upward speed did you throw the stone?

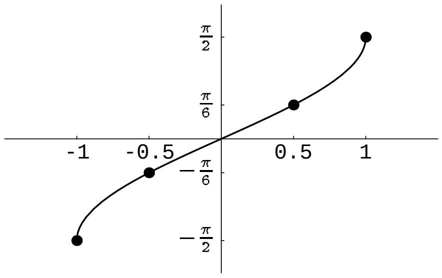
(b) If your stopwatch is accurate only to ± 0.1 sec, how accurate is your answer to part (a)? Use the tangent line approximation.

5. (12 points) You have a 1000-square-foot apartment which you're renting for the summer to someone from Europe, so you have to convert its area to square meters in order to explain its size. You know that there are about 10 square feet in one square meter. More precisely, 1 square meter = 10.77 square feet. Use the tangent line approximation to find the area of your apartment in square meters. You do **NOT** need a calculator to do this problem correctly.

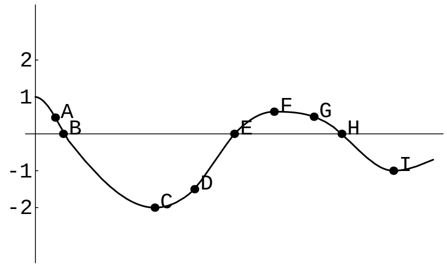
Practice First Midterm #3 (50 points in all, time = 1 hour)

1. (8 points) The graph of $y = f(x)$ is pictured below to the left. It is called the "arcsine" function, and its domain is the interval $[-1, 1]$. Find the domain of

$$y = \frac{1}{\sqrt{\frac{\pi}{6} + f(x-1)}}.$$



Graph for Problem 1



Graph for Problem 2

2. (12 points) At time $t = 0$ an object starts from 0 moving to the right at 1 m/sec. Its velocity changes, as shown in the graph above at the right. This graph shows the velocity with which the object is moving to the right as a function of time. Give the letters of **ALL** labeled points where

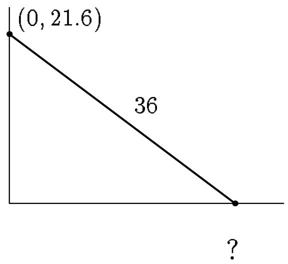
(a) the object is actually moving rightward;
(b) the object is actually moving leftward;
(c) the object has positive (rightward) acceleration;
(d) the object has zero acceleration;
(e) the object's speed heading to the left is maximum.

3. (15 points) In this problem use the formula $s = \frac{1}{2}gt^2$ for the distance an object falls in t seconds (with no air resistance). Upon arrival on a new airless planet, you decide to measure its gravitational acceleration g by dropping a stone from a height of **exactly** 11.25 meters and measuring the time t it takes to fall to the ground.

(a) If you time 3.00 sec for the stone to reach the ground, what is g?

(b) If you do not know the time precisely, but only know that it is between 3.00 and 3.10 sec, what is the range you know g to be in? Use the tangent line approximation.

4. (15 points) You lean a yardstick (exactly 36 inches long) against a wall, and measure the height up the wall to where the yardstick touches to be 21.6 inches $\pm$ 0.2 inches. Find the distance the other end of the yardstick is from the bottom of the wall (see the diagram to the right). Include the error, which you find using the tangent line approximation.



Practice First Midterm #4 (50 points in all — time = 1 hour)

1. (10 points) Let $f(x) = \sqrt{625 - x^2}$. Find the domain of each of the following functions:

$$\text{(a)} \ \frac{x}{5f(x+10)}, \qquad \text{(b)} \ \sqrt{f(x) - 24}.$$

2. (15 points) The downward velocity of a parachutist measured in m/sec is given by the graph at the bottom of the page on the left.

(a) Sketch the graph of downward acceleration.

(b) At what instant is the downward velocity the greatest? What (in practical terms) is happening at that instant? At what instant does the acceleration have the greatest positive value? What is happening at that instant? At what instant does the acceleration have the greatest negative value?

3. (10 points) Sketch the circle $y = \pm\sqrt{100 - x^2}$ and the ellipse $y = \pm 2\sqrt{100 - x^2}$. The point $(8, 12)$ is on the latter ellipse. Use the tangent line approximation to find the y-coordinate of the point on the ellipse in the first quadrant whose x-coordinate is 7.7. You *must* use the tangent line approximation in this problem, showing your work clearly.

4. (15 points) A game board is to be constructed in the shape of a rectangle of dimensions $r \times s$ with a half-circle attached to each of the two sides of length r (see the bottom of this page at the right).

(a) Find a formula for the area of the board (rectangle plus two half-circles) in terms of r and s.

(b) You first construct a board having dimensions $r = 20$ cm, $s = 30$ cm. What is the area of the board?

(c) Now you want to increase r to 21 cm. What must you do to s so that the total area of the board remains the same? Use the tangent line approximation. Be sure to show clearly what the function is to which you're applying the tangent line approximation formula.