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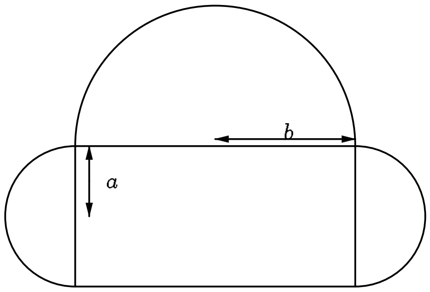
Practice Second Midterms

Practice Second Midterm #1 (50 points in all, time = 1 hour)



1. (13 points) A rectangle having dimensions $2a$ and $2b$ has semicircles on three sides, as shown in the drawing at the right. Suppose that at first $a = 10$ and $b = 20$, but then you want to increase a to 10.100 and decrease b in such a way that the total area of the shape (the rectangle plus the three semicircles) remains constant. Use the tangent line approximation to determine your new value for b.

2. (12 points) Let $y = f(x) = x - \big(1/(2x^2)\big)$.

(a) Find the x- and y-coordinates of any upper and lower turning points, and **use the second derivative test** to determine if they are max or min.

(b) Find all intervals of values of x where the function is **increasing**.

(c) Find all vertical asymptotes (if there are any).

(d) Find all points of inflection (if there are any), and determine the upward or downward concavity of each part of the curve.

(e) Sketch the curve. Show in your sketch what $f(x)$ looks like as x approaches $+\infty$ and $-\infty$, and as x approaches the vertical asymptote (if there is one) from the left and from the right.

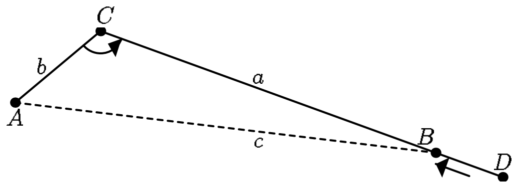
3. (10 points) Suppose that in Wanganui, New Zealand the mean daily temperature (averaged over many years) is a sinusoidal function of time. Let t be the time **in months** from the beginning of the year, and let y be the temperature in °F. Suppose that the hottest temperature is 75°F in mid-January (that is, $t = 0.5$), and the coldest is 35°F in mid-July.

(a) Graph the temperature function $y = f(t)$.

(b) Give its equation, in the form $y = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$.

(c) Suppose that the typical Wanganuian homeowner turns on the heat when the mean daily temperature drops below 65°F. In an average year, find the time interval during the year when she has the heat turned on.

4. (15 points) According to the law of cosines for the triangle ABC below, we have: $c^2 = a^2 + b^2 - 2ab\cos C$. Suppose that $C = 120°$, $b = 50$ meters, and you are running from D to C at 5.70 m/sec. How fast is your distance c from A decreasing when you are at a distance $a = 160$ m from C?

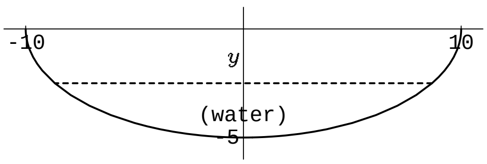


Practice Second Midterm #2 (50 points in all, time = 1 hour)

1. (13 points) Let $y = f(x) = x^2 - \frac{1}{6}x^4$.
(a) Find the x- and y-coordinates of any upper and lower turning points.
(b) Find all intervals of values of x where the function is **increasing**.
(c) Find all points of inflection.
(d) For what values of x is the function concave **upward**?
(e) Sketch the curve. Show in your sketch what $f(x)$ does as x approaches $+\infty$ and $-\infty$.

2. (12 points) A bicycle has a wheel whose **circumference** is exactly 2 meters. It is traveling at 5 m/sec from left to right. At time $t = 0$ it picks up a pebble in the tread of the wheel. Take $(0, 0)$ to be the point where the pebble attaches itself to the tread. In this problem use the parametric equations of the cycloid: $x = r\theta - r\sin\theta$, $y = r - r\cos\theta$.
(a) What is the angular velocity $d\theta/dt$ of the wheel in rad/sec?
(b) Find formulas for the vertical velocity and the horizontal velocity of the pebble in terms of θ.
(c) Find the slope of the direction of motion of the pebble when $t = 1/3$ sec. Draw a picture of the wheel showing the position of the pebble when $t = 1/3$ sec, and draw an arrow showing the direction of motion of the pebble when $t = 1/3$ sec.

3. (12 points) (a) Write the implicit equation of the ellipse whose bottom half is pictured below to the right, and solve for x in terms of y.
(b) Suppose that the drawing is the side view of a container which has a circular top (and circular cross-section at any level). That is, the container is obtained by rotating the bottom half of the ellipse around the y-axis. Suppose the container is being filled with water. Let $A = f(y)$ be the **surface area** of the top of the water when it reaches the level y on the y-axis. The units are **meters**. Write a formula for A in term of y.

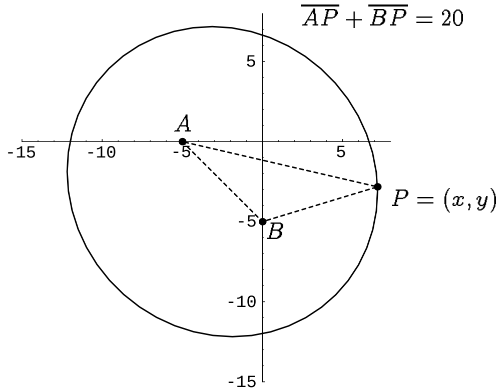


(c) If the water is 250 cm = 2.5 m deep at its deepest point and is rising at 1 cm/sec, how fast is the surface area of the top of the water increasing?

4. (13 points) The ellipse pictured to the right has foci at the two points $(-5, 0)$ and $(0, -5)$. The distance of any point (x, y) on the ellipse from $(-5, 0)$ plus its distance from $(0, -5)$ has the constant value 20.

(a) Write an equation for this ellipse in the form $F(x, y) = \text{Const}$.

(b) Notice that the point $(0, -12)$ is on the ellipse. Find the slope of the tangent line to the ellipse at $(0, -12)$.



Practice Second Midterm #3 (50 points in all, time = 1 hour)

1. (12 points) Let $y = f(x) = 0.25\,x^4 - 2\,x^3 + 4.5\,x^2$.

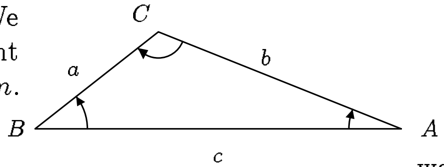
(a) Find the x- and y-coordinates of any upper and lower turning points.

(b) Find all points of inflection.

(c) Sketch the curve, labeling all of the points found in parts (a) and (b) ("max," "min," "inflec").

(d) Give the exact interval(s) where the curve is concave downward.

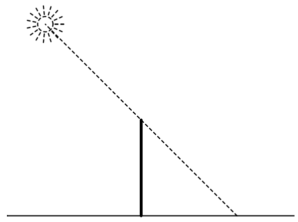
2. (12 points) In the triangle shown below, the sides a and b are pieces of metal hinged at C. Here a is **exactly** $1.50\,m$ long and b is **exactly** $2.50\,m$ long. We measure the side c to be $3.50\,m$, but our measurement of c has an error of up to a centimeter, i.e., $\pm 0.01\,m$. Using the law of cosines



$$c^2 = a^2 + b^2 - 2ab\cos C,$$

we find that the angle C is 120°. Use the tangent line approximation and implicit differentiation to determine the error **in degrees** (not radians) in our value for the angle C.

3. (12 points) You are 5 ft tall, and the sun is casting a 5 ft shadow. (See the diagram at the right.) If the sun is descending toward the horizontal at the rate of 1/4 degree per minute, how fast is your shadow lengthening? (First express the length of your shadow as a function of the sun's angle above the horizontal.)



4. (14 points) An object oscillates horizontally at the end of a spring. Its sinusoidal horizontal motion goes from a minimum position of $x = 0$ to a maximum position of $x = 4$ m. It goes back and forth once every 2 seconds, starting at $x = 0$ at time $t = 0$. Meanwhile, at time $t = 0$ the whole spring system is dropped from a height of 19.6 meters. Neglect air resistance, and take $g = 9.8$ m/sec^2.

(a) Write a formula for $y(t)$.

(b) Draw a graph of x as a function of t, and find a formula for $x(t)$.

(c) At the instant $t = 1.25$ sec, find the horizontal and vertical components of velocity, and the direction (slope) of the path.

Practice Second Midterm #4 (50 points in all, time = 1 hour)

1. (15 points) Let $y = f(x) = \frac{1}{120}\left(2x^6 - 9x^4 - 108x^2\right)$.

(a) Find the x- and y-coordinates of any upper and lower turning points.

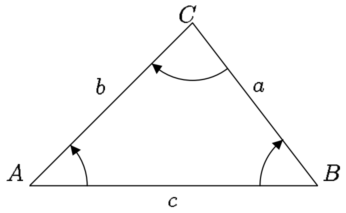
(b) Find all intervals of values of x where the function is **increasing**.

(c) Find all points of inflection.

(d) For what values of x is the function concave **downward**?

(e) Sketch the curve. Show in your sketch what $f(x)$ does as x approaches $+\infty$ and $-\infty$.

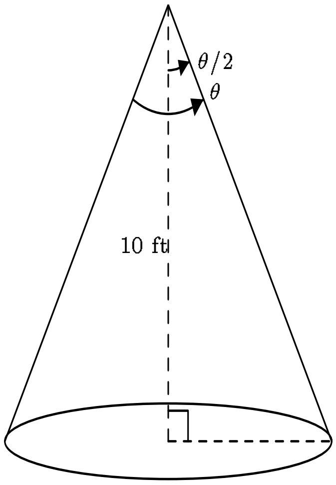
2. (10 points) A circular oil slick of uniform thickness is caused by a spill of 1 cubic meter of oil. The thickness of the oil is **decreasing** at the rate of 0.1 cm/hr as the oil spreads (note: 1 cm = 0.01 m). At what rate is the radius of the slick increasing when it is 8 meters? (Note: the volume of a circular oil slick is equal to $\pi\times(\text{radius})^2\times$thickness.)



3. (10 points) In the triangle pictured at the right, let A, B, C be the angles at the three vertices, and let a, b, c be the sides opposite those angles. According to the "law of sines," you always have:

$$\frac{b}{a} = \frac{\sin B}{\sin A}.$$

Suppose that a and b are pieces of metal which are hinged at C. At first the angle A is 45° and the angle B is 60°. You then widen A to 46°, without changing the sides a and b. What happens to the angle B? Use the tangent line approximation.



4. (15 points) A conical pile of sand is exactly 10 ft high. Suppose that you have an instrument to measure the vertex angle θ (see the diagram at the right).

(a) Express the volume V of sand in terms of θ.

(b) Suppose you measure θ to be 60° with an error of $\pm 1°$. Find V, and use the tangent line approximation to determine the error in your value for V. (Warning: use radians for angles.)