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Laboratory Calculus I: Project

Forces on a Piston

Date: ____________________

Team Members:

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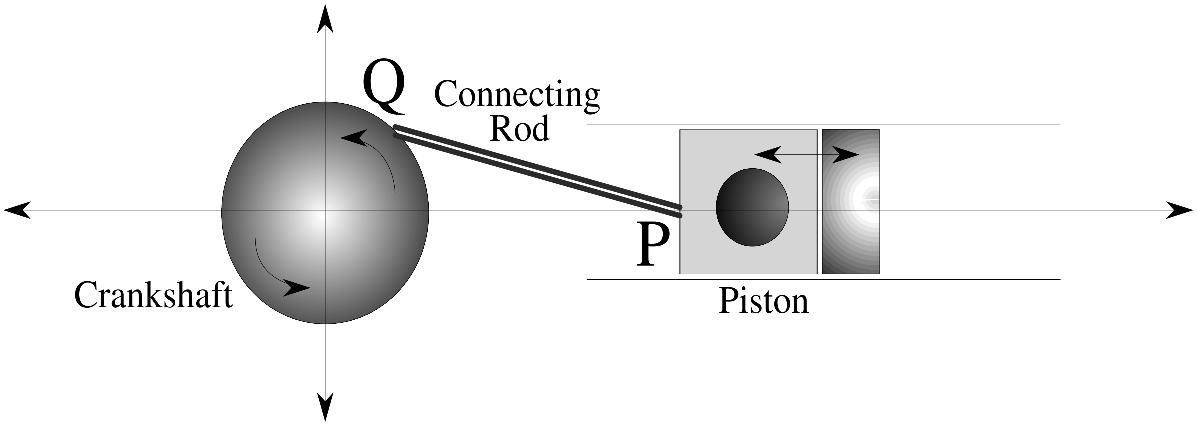
The problem: In an internal combustion engine, each piston is housed in a cylinder and is attached to the crankshaft by a connecting rod as indicated in the diagram below. The piston moves back and forth in the cylinder, and, in response the crankshaft rotates. When the crankshaft is turning, even at a constant angular velocity the (linear) velocity and speed of the piston are constantly changing. Investigate the effect of the length of the connecting rod on the velocity and acceleration of the piston.



1. Suppose the crank is 1.5 inches in radius and the connecting rod is 6 inches long and the crank is turning at a constant 1 revolution per second. Assume the piston is moving on the x-axis and express the position of the point at which the rod connects to the piston (the wrist pin) as a function of t (in seconds).

   Hint: Let r denote the crank radius, l the length of the connecting rod, $\theta(t)$ the crank angle at time t and assume $\theta(0) = 0$ and that the rotation is counterclockwise. We can place the center of the crank at the origin. Then if $x(t)$ is the position of the wrist pin at time t, the law of cosines says that

   $$l(t)^2 = r^2 + x(t)^2 - 1rx(t)\cos(\theta(t)).$$

   Solve for $x(t)$ and choose one of the solutions (e.g. the one for which $x(t) = r + l$ when $t = 0$).
2. Calculate expressions for the position, velocity, and acceleration of the wrist pin as a function of time and plot all three on the same graph for t from 0 to 2 seconds. At what times are the velocity and acceleration maximal and minimal? What are the maximum velocity and acceleration?
3. Repeat the above calculations for various lengths of the connecting rod from 2 to 8 and observe the effect of these changes on the velocity and acceleration of the wrist pin. Note particularly the change in shape of the acceleration curve and formulate a conjecture as to length of the connecting rod for which marks the transition from one shape to the other. Develop additional evidence for your conjecture through additional experiments.