Answers

1. Answers to Selected Homework Problems

Homework 1

1. (a) $8\sqrt{2}/15 = 0.754$, (b) $1/12$, (c) $(n-1)/(n+1)$, (d) 4.5 (the curves intersect at $(-3,2)$ and $(0,-1)$), (e) $4/3$, (f) $(2/3)-(2/\pi) = 0.030$, (g) $(3/\pi)(1-0.5\sqrt{3})-(1/8) = 0.00294$.
2. (a) $\sqrt{2}-1 = 0.414$, (b) $46/3$.
3. (a) net$=1/\pi$, total$=5/\pi$, (b) net=0, total=245, (c) net=20, total=28, (d) net$=(1.5/\pi)-(1/2) = -0.0225$, total$=0.25+(4.5-3\sqrt{3})/\pi = 0.0284$.
4. (a) $a_x(t) = (F/m)\cos(\omega t)$, $a_y(t) = (F/m)\sin(\omega t)$, (b) $v_x(t) = (F/m\omega)\sin(\omega t)$, $v_y(t) = (F/m\omega)(1-\cos(\omega t))$, (c) $x(t) = \left(F/m\omega^2\right)(1-\cos(\omega t))$, $y(t) = \left(F/m\omega^2\right)(\omega t - \sin(\omega t))$, (d) a cycloid running up the y-axis (rather than the x-axis), (e) a circle if $v_0 = -F/m\omega$.

Homework 2

1. hits ground at (a) 3 sec, (b) 4.5 sec, (c) 2 sec. 2. 44.3 m/sec, 0.232 sec.
3. $v_0 = -38$ ft/sec, $s_0 = 590$ ft. 4. 1032 ft.
5. (a) $8\pi/3$, (b) $\pi/30$, (c) $0.5\pi^2 - \pi = 1.793$. 6. (a) 22.8π, (b) 14.8π, (c) 20π, (d) 4π.
7. (a) $\frac{4}{3}\pi ab^2$, (b) $\frac{4}{3}\pi a^2 b$. 8. $\pi h^2\left(r-\frac{1}{3}h\right)$. 9. $\int_0^H \frac{\sqrt{3}}{4}(\frac{x}{H}s)^2 dx = \frac{\sqrt{3}}{12}s^2 H$.

Homework 3

1. (a) 6 min 17 sec, (b) 0.03183 units/min. 2. (a) 20 min 57 sec, (b) 0.03183 units/min. 3. 0.6283 ft^3/min. 4. $dV/dt = 0.04\pi\left(4-(-2+0.04t)^2\right)$. 5. $dV/dt = 0.09\pi\left(4-(-2+0.04t)^2\right)$. 6. 1.21 cm/sec. 7. cross-sectional area $= \frac{1}{4}\sqrt{3}(\frac{h}{H}s)^2$.

Homework 4

1. (a) $x^2/4+4y^2 = 1$, (b) $x^2/100+y^2/64 = 1$, (c) $x^2/625+y^2/576 = 1$, (d) $x^2+y^2/9 = 1$, (e) $x^2/16+y^2/25 = 1$.
2. (a) $(\pm 3, 0)$, $e = 0.6$; (b) $(0, \pm 24)$, $e = 0.96$; (c) $(\pm 0.3727, 0)$, $e = 0.7454$. 3. 10.
4. (a) $\int_0^b \sqrt{1+n^2x^{2(n-1)}}\,dx$; (b) $\frac{8}{27}\left((1+9b/4)^{3/2}-1\right)$. 5. (a) $\int_0^4 \sqrt{98+(-9.8t+7\sqrt{2})^2}\,dt$; (b) $\int_0^{\pi/2}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}\,d\theta$; (c) r$\int_0^{2\pi}\sqrt{1.25-\cos\theta}\,d\theta$. 6. (a) $1/\sqrt{7} = 0.3780$; (b) $\frac{d}{dt}(x+2.5y) = 0.3780(1+2.5\sqrt{3}/2) = 1.1964$; (c) $u = x + y\cot\alpha$, so $\frac{du}{dt} = \frac{dx}{dt} + \frac{dy}{dt}\cot\alpha + y(-\csc^2\alpha)\frac{d\alpha}{dt} = 1.1964 - y(\csc^2\alpha)\left(-15\cdot\frac{\pi}{180}\right) = 1.1964 + 0.5\cdot 7.2509\cdot\pi/12 = 2.1455$ units/hour.

Homework 5

1. 55.4 m/sec 2. (a) 0.790766, (b) 1.183465, (c) 0.987116, (d) 0.997943. 3. 8.878.
4. (a) 0.58726, (b) 0.58622 (limits of integration are $\pi/3$ and $\pi/2$).
5. (b) 0.2402 m, (c) 0.4541 m.
6. (a) $x(8)-x(0) = \int_0^8 \dot{x}(t)dt \approx 34.45$, $y(8)-y(0) = \int_0^8 \dot{y}(t)dt \approx 46.35$, and finally $\sqrt{34.45^2+46.35^2} = 57.75$; (b) $\int_0^8\sqrt{(dx/dt)^2+(dy/dt)^2}dt \approx 59.29$.

Homework 6

1. (a) (2,1), (b) (1.5,1.2), (c) $(0.4244r, 0)$, (d) $(0, -0.8377)$.

2. at 39.37° northern latitude & 116.6° western longitude (right between Summit Mountain and Hickison Summit on the border between Lander and Eureka counties). 3. 0.5466.

Homework 7

1. (a) $2x^2/(4+x^2) + 2x \arctan(x/2)$.
(b) $\left(\arcsin(x^2) - 2x(x+1)(1-x^4)^{-1/2}\right) \Big/ \left(\arcsin(x^2)\right)^2$.
(c) $-1/2\sqrt{x-x^2}$, (d) $\arcsin x$. 2. (a) $0.25 \arcsin(0.4x) + C$, (b) $0.5\sqrt{2} \arctan(\sqrt{2}\,x) + C$. 3. 31.32°.
4. $\pi^2/4$. 5. (0.2559,0).
6. (a) max: $(1, 0.2854)$, min: $(-1, -0.2854)$, (b) max: $(-\sqrt{3}, 0.685)$, min: $(\sqrt{3}, -0.685)$.
7. (a) $\pi - (2/3)$, (b) $2\sqrt{3} - 2\pi/3$. 8. 4.584 deg/sec.
9. (a) $\arccos\left((a^2+b^2-x^2)/2ab\right)$, (b) $2c \,/\, \sqrt{(2ab)^2-(a^2+b^2-c^2)^2}$.

Homework 8

1. (a) $1/x$, (b) $(4\ln^3 x)/x$, (c) $(2\ln(\ln x))/(x\ln x)$, (d) $1/(\sin x \cos x)$, (e) $-\arctan(x/4)$.
2. (a) both derivatives are 0; (b) use the fact that only a constant function has derivative zero, find the constant by setting $x = 1$, and then replace x by b in the first function and by a in the second function.
3. (a) $\ln 3$, (b) $\frac{3}{2}\ln 2$, (c) $\frac{2}{3}\ln 3$, (d) $\frac{3}{2}\ln 3 - \ln 2$, (e) $\ln \pi$, (f) $\frac{1}{2}\ln(r+x)$.
4. (a) $-\ln 2$, (b) $\ln \sin x + C$, (c) $\ln(\ln x) + C$, (d) $(1/\ln 2) - (1/\ln 3) = 0.5325$, (e) $2\ln 3$.
5. (a) 1.4427, (b) 1.3863, (c) 0.4413. 6. (a) 0.4449, (b) 0.09436. 7. 4.3552.
8. 11.18, 14.14, 22.36.
9. (a) 0, (b) 0, (c) 0, (d) 0. 10. (a) $(1+\ln x)x^x$, (b) $\left(\sqrt{x}\cot x + (\ln\sin x)/2\sqrt{x}\right)(\sin x)^{\sqrt{x}}$,
(c) $\left(\frac{1}{\ln x} + \ln(\ln x)\right)(\ln x)^x$, (d) $\left(\frac{2}{x}\ln x\right) x^{\ln x}$, (e) $\left(-(\arctan x)\tan x + \frac{\ln\cos x}{1+x^2}\right)(\cos x)^{\arctan x}$.
11. Let a be the number of hours before noon when it started snowing, and let $s(t)$ be the distance traveled after t hours of plowing. Show that ds/dt is inversely proportional to $t+a$, and then use the method in section 2 of Chapter 1 to find a formula for $s(t)$; you should find that $s(t) = C\ln((t+a)/a)$. (The value of the constant C makes no difference.) You're given that $s(2) = \frac{3}{2}s(1)$. Use the properties of ln and some algebra to get a quadratic equation for a, from which you find that $a = (-1+\sqrt{5})/2$ (this number is called the "golden ratio"). Snow started falling at 11:23 a.m.

Homework 9

1. (a) 2, (b) ∞, (c) ∞, (d) 2, (e) $\pi/2$, (f) π, (g) $\pi/3$, (h) $\pi/2$, (i) π, (j) 1, (k) ∞, (l) $\pi/4$.
2. $\pi n/(2-n)$, infinite volume if $n \geq 2$. 3. $(n-1)/(n-2)$.

Homework 10

1. (a) 1, (b) 0.9917. 2. (a) $3x$, (b) $-x^2$, (c) x/y^2, (d) x, (e) $\frac{1}{2}x^2$. 3. (a) $20\ln x$, (b) x^2+x,
(c) $\arcsin e^x$, (d) $\pm\sqrt{2\ln x}$, (e) $1/(1+e^{0.01x})$. 4. (a) $e^{2x}\left(\frac{1}{1+x^2} + 2\arctan x\right)$,
(b) $e^{-\sin(\pi x)}(1 - \pi x\cos(\pi x))$, (c) e^{x+e^x}, (d) $e^{-\cos x}(2x + x^2\sin x)$, (e) $e^{\arcsin x}/\sqrt{1-x^2}$. 5. (a) $\frac{d}{dx}\cosh x = \sinh x$, $\frac{d}{dx}\sinh x = \cosh x$, (b) 1.
6. (a) 3.6, (b) $\frac{1}{2} - \frac{1}{2e}$, (c) $e - \frac{1}{e}$, (d) $1/\ln 10$. 7. $\frac{15\pi}{16\ln 4} \approx 2.1245$.
8. (a) max at $(\pi/6, 1.9477)$, inflection points at $(\pi/18, 1.3956)$ and $(5\pi/18, 1.3956)$.
(b) max at $(0.6065, 0.1839)$, inflection at $(0.2231, 0.0747)$.

Homework 11

1. \$4932. 2. (a) 11.55 yrs, (b) 6.93 yrs, (c) 3.85 yrs; (a) 11.90 years, (b) 7.27 yrs, (c) 4.19 yrs. 3. 20480.
4. 1020201, 1491825, 34.66 yrs. 5. $250e^{0.007675(t-1650)}$, A.D. 2250. 6. 11.33%, 6.12 yrs. 7. 6.05 times
8. (a) \$4034300, (b) $100e^{0.13t}$, (c) $\int_0^{10} 52 \cdot 100e^{0.13t} = 40000e^{0.13t}|_0^{10} = \$106772$, (d) \$44241000, (e) about 2/3 day. 9. $8033\ln(y_0/y_1)$. 10. 6.64 hrs. 11. $100e^{-0.0112t}$, 62.1 yrs. 12. \$11477. 13. (a) $v(t) = v_0e^{-kt}$; (b) $s(t) = \frac{v_0}{k}(1-e^{-kt})$; (c)$\arctan(ky_0/v_0)$.

Homework 12

1. 2.5587. 2. (a) $-\frac{1}{\pi}\sqrt{2+\cos(2\pi t)} + C$, (b) $-1/\left(7a\sin^7(ax+b)\right) + C$. 3. 2.0191.
4. (a) $4/(3\pi) = 0.4244$, (b) $3/8 = 0.375$, (c) $16/(15\pi) = 0.3395$, (d) $5/16 = 0.3125$, (e) $32/(35\pi) = 0.2910$.

Homework 13

1. (a) $-\frac{x}{a}\cos(ax+b)+\frac{1}{a^2}\sin(ax+b)+C$, (b) $x^2\sin x+2x\cos x-2\sin x+C$,
(c) x arctan $x-\frac{1}{2}\ln\left(1+x^2\right)+C$, (d) $\left(x^3-3x^2+6x-6\right)e^x+C$,
(e) $\frac{1}{2}x^2$ arctan $x-\frac{x}{2}+\frac{1}{2}$ arctan $x+C$.
2. (a) $\left(-\frac{1}{k},\frac{1}{4}\right)$, (b) $(9/16,0)$, (c) $\left(\left(\frac{n+1}{n+2}\right)^2,0\right)$, (d) 0.18484. 3. 0.60672.

Homework 14

1. (a) 0.2747, (b) 0.09861, (c) 0.05371, (d) 0.4621, (e) 0.06112, (f) 0.04251, (g) 0.2118, (h) 0.2, (i) 0.1903, (j) $-\frac{1}{5}\ln(6)=-0.35835$.
2. (a) (0.7095,0), (b) (0.7912,0). 3. $\frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}+C$.

Homework 15

1. $\arcsin(x/r)+C$. 2. (a) $\ln|x+\sqrt{x^2+r^2}|+C$, (b) $\ln|x+\sqrt{x^2-r^2}|+C$. 3. $(0.37616,0)$. 4. 0.93313. 5. 0.48121.

Homework 16

1. \$27.98, \$1007.30. 2. (a) \$379.15, \$27749, (b) 7.7 years, (c) \$2613.91 (1st year), \$1574.15 (5th year). 3. \$48633.80, \$9726.76. 4. (a) \$5613.14, (b) only \$17.01 more, (c) 22.57 years.
5. $T-(\ln 2)/k=13$ years and 8.22 months.
6. (a) \$44754, (b) \$328.12, (c) 64.9 years, (d) \$298.36. 8. 9:43 p.m. 9. 7.27 hours
10. 4:58 a.m. 11. 8:57 a.m. 12. (a) 0.2, (c) 40.3 m/sec upward, (d) 1.71 sec.
13. $s(t)=\begin{cases}98t+980\left(e^{-0.1t}-1\right), & \text{for } 0\le t\le 10;\\ 7t-39.25e^{-1.4(t-10)}+329.77, & \text{for } t>10,\end{cases}$ 24.3 sec.
14. $v(t)=\begin{cases}9.8t, & \text{for } 0\le t\le 7;\\ 19.6+49.0e^{-0.5(t-7)}, & \text{for } 7\le t\le 15.\end{cases}$
15. (a) $y=10-5e^{-0.03t}$, (b) $y=10+10e^{-0.03t}$, (c) $y=10$. 16. 7.42 lb.
17. (a) $y(t)=48\left(1-e^{-t/12000}\right)$, (b) 36 min.
18. (c) $Vk+VP/r$, (e) Superior 431 yr, Michigan 71.4 yr, Erie 6.05 yr, Ontario 17.6 yr.
19. (a) $dy/dt=-ky+I$, (b) 339 million, 400 million. 20. $R=80565$ per year; if $R=300000$, then $y(t)=3147600-147600e^{0.0953t}$, and the salmon will die out after 32.1 years.

Homework 17

1. (a) $y=\sqrt{3x/(6+x)}$, (b) $y=1/(10-\frac{1}{3}x^3)$, (c) $y=\sqrt{\frac{2}{\pi}\sin(\pi x)\ +\ 1}$.
2. (a) $y(t)=M/\left(1+Ce^{-kt}\right)$, (b) $100/\left(1+19e^{-0.9474t}\right)$, 6.216 years.
3. $y_0(1-0.02t)^{2.5}$, domain is $0\le t\le 50$. 4. (a) $v_\infty=49$ m/sec, (b) $dv/(100+v)(49-v)=0.002dt$, then get $(100+v)/(49-v)=e^Ce^{0.298t}$, where $e^C=100/49$ (since $v(0)=0$), finally end up with $v(t)=49-\left(73.01/\left(e^{0.298t}+0.49\right)\right)$, (c) $\int_0^3 49-\left(73.01/\left(e^{0.298t}+0.49\right)\right)$.
(d) $ds/dv=v/\left(9.8-0.102v-0.002v^2\right)$, so $\int_0^{48}v\,dv/\left(9.8-0.102v-0.002v^2\right)=508$ m.
5. 205 m. 6. (a) $\frac{dy}{dt}=-\frac{r(t)}{V}\,y$, (b) $y_0e^{-0.02t-0.0025t^2}$, (c) $y_0e^{-0.01t+\frac{0.01}{\pi}\cos(\pi(t-0.5))}$.
7. $dy/dt=-2y/(100+0.25t^2)$, $y(t)=y_0e^{-0.4\ \arctan(0.05t)}$, $y(\infty)=y_0e^{-0.2\pi}=0.5335y_0$.
8. (a) $\frac{dy}{dt}=-\frac{r(t)}{V}(y-kV)$, where $V=100$. (b) $y(t)=100k+(y_0-100k)e^{-0.01\int_0^t r(u)\,du}$.
(c) $\int_0^{10}r(u)du=26.73$; $e^{-0.01\cdot 26.74}=0.765$, i.e., 23.5% decrease.
(d) $y(10)=40+(30-40)e^{-0.01\cdot 26.74}=32.35$ gm.
9. (a) $\frac{dv}{dt}=-8-0.4v-0.01v^2$, (b) $v(t)=20\left(\tan(1.19-0.2t)-1\right)$, (c) 2.024 sec.
10. (a) $(m-\alpha t)\frac{dv}{dt}=F-Kv$, (b) $v(t)=\frac{F}{K}+\left(v_0-\frac{F}{K}\right)\left(1-\frac{\alpha t}{m}\right)^{K/\alpha}$.
11. $y(t)=y_0\sin(\omega t+\frac{\pi}{2})=y_0\cos(\omega t)$. 12. $y(t)=\frac{v_0}{\omega}\sin(\omega t)+y_0\cos(\omega t)$.

Homework 18

2. (a) $y_0=D-Ak\alpha/(k^2+\alpha^2)$, (b) for k small the time delay is nearly 6 hours and the amplitude is small, while for k large the time delay is small and the amplitude is only a little less than A, (c)

for k near zero, time delay$=\frac{1}{\alpha}\left(\pi/2-\arctan\left(\frac{k}{\alpha}\right)\right)\approx 6-k/\alpha^2=6-144k/\pi^2$ (since $\arctan(x)\approx x$ if x is near 0), while for k large, time delay$=\frac{12}{\pi}\arctan\left(\frac{\pi}{12k}\right)\approx 1/k$, (d) $\theta=60°$, so time delay is 4 hrs, hottest: 7 p.m.
3. $dy/dt=-0.01y+(2+0.25t)$. 4. $\frac{dy}{dt}=-k\big(y-(b+at)\big)$.
5. $dy/dt=-2y/(100+0.25t^2)+(4+t)$. 6. $dy/dt=-\frac{r}{V}y+r(k+\alpha t)$.
7. (a) $dy/dt=ky+\left(18.5\sin\left(\frac{2\pi t}{365}\right)-20\right)$, (b) \$29200, (c) you can use the solution to Example 18.2 by replacing k by $-k$ throughout Example 18.2, where $k=0.00025\,\mathrm{day}^{-1}$, and choosing $\alpha=-2\pi/365$, $\theta=\arctan(2\pi/365k)=1.5563\,\mathrm{rad}$, $D=80000$, $A=7400$; then $y(t)=80000+(y_0-78925.5)e^{0.00025t}-1074.58\sin\left(\frac{2\pi}{365}t+1.5563\right)$; (d) \$24136.
8. (a) $r(t)=12-at$, $V(t)=100-12t+\frac{1}{2}at^2$; (b) $\frac{dy}{dt}=-\frac{r(t)}{V(t)}y+l$.
9. (a) $x_0e^{-k_1t}$, (b) $dy/dt=-k_2y+k_1x_0e^{-k_1t}$.
10. (a) $x(t)=kV+(x_0-kV)e^{-rt/V}$, (b) $dy/dt=-\frac{r}{W}y+\left(rk+\left(\frac{x_0}{V}-k\right)re^{-rt/V}\right)$.

2. Answers to Review Problem Sets

Answers to Review Problem Set #1: First Midterm

1. $v(t)=-1.5+\int_0^t\sin(u/3)du=1.5-3\cos(t/3)$; velocity is at first negative, then becomes positive after $v(t)$ crosses the t-axis, i.e., when: $1.5-3\cos(t/3)=0$; $\cos(t/3)=0.5$; $t/3=\pi/3$; $t=\pi$. So total distance $=-\int_0^\pi(1.5-3\cos(t/3))dt+\int_\pi^5(1.5-3\cos(t/3))dt==-(1.5t-9\sin(t/3))|_0^\pi+(1.5t-9\sin(t/3))|_\pi^5=-1.5\pi+9(\sqrt{3}/2)+7.5-9\sin(5/3)-(1.5\pi-9(\sqrt{3}/2))=-3\pi+9\sqrt{3}+7.5-9\cdot 0.9954=4.705$.
2.
$$\text{(a) } \pi\int_0^{1/2}\cos^2(\pi x)-(1-2x)^2dx; \qquad \text{(b) } \overline{x}=0,\ \overline{y}=\frac{\int_0^4 1.5x^2dx+\int_4^5 2x\sqrt{25-x^2}dx}{\int_0^4 1.5x\,dx+\int_4^5 2\sqrt{25-x^2}dx}.$$
3. (a) $\frac{1}{5}\cdot\pi\int_{-3}^0 x^2dy=\frac{\pi}{5}\int_{-3}^0 25(1-y^2/9)dy=5\pi(y-y^3/27)|_{-3}^0=10\pi$ sec.
(b) $5/(\text{cross-sectional area})=5/(\pi 5^2)=0.2/\pi$ units/sec; the average rate is total rise divided by T, i.e., $3/10\pi=0.3/\pi$, which is 1 1/2 times as great.

4. (a) $r\int_0^\pi\sqrt{(1-0.5\cos\theta)^2+(0.5\sin\theta)^2}\,d\theta=r\int_0^\pi\sqrt{1.25-\cos\theta}\,d\theta$.
(b) $r\frac{\pi}{6}\Big(\frac{1}{2}\cdot 0.5+\sqrt{1.25-0.5\sqrt{3}}+\sqrt{1.25-0.5}+\sqrt{1.25}+\sqrt{1.25+0.5}+\sqrt{1.25+0.5\sqrt{3}}$
$+\frac{1}{2}\cdot 1.5\Big)=r\frac{\pi}{6}(0.25+0.6197+0.8660+1.1180+1.3229+1.4547+0.75)=3.3412r$.

Answers to Review Problem Set #2: First Midterm

1. $x(t)=(10\cos\alpha)t$, $y(t)=-4.9t^2-(10\sin\alpha)t+s_0$. Since $y(5)=0$ and $x(5)=25$, from the second equation we get $\cos\alpha=0.5$, i.e., $\alpha=60°$, and from the first equation we then get $0=-4.9\cdot 5^2-(10\cdot 0.5\sqrt{3})5+s_0$, so that $s_0=165.8$ m.
2.
$$\text{(a) } \pi\int_0^2\frac{1}{3-y}dy \qquad \text{(b) } \int_0^{0.1}5\sqrt{(1-\cos(12.5t))^2+\sin^2(12.5t)}\,dt.$$
3. (a) Taking d/dt of $\frac{x^2}{25}+\frac{y^2}{16}=1$ gives $y\,\dot{y}=-0.64x\,\dot{x}=-0.384$. When $x=3$, we have $y=3.2$, so that $\dot{y}=-0.384/3.2=-0.12$. (b) The position of the shadow is $x+y$, so that its rate is $\dot{x}+\dot{y}=0.20-0.12=0.08$. (c) $\sqrt{\dot{x}^2+\dot{y}^2}=\sqrt{0.20^2+0.12^2}=0.2332$.

4. (a) $\int_0^\pi\sin(x)dx\Big/\int_0^\pi\frac{\sin x}{x}dx$. Top integral $=2$.
(b) Bottom integral $=\frac{\pi}{6}\left(\frac{1}{2}\cdot 1+\frac{0.5}{\pi/6}+\frac{0.5\sqrt{3}}{\pi/3}+\frac{1}{\pi/2}+\frac{0.5\sqrt{3}}{2\pi/3}+\frac{0.5}{5\pi/6}+0\right)=1.84465$.
(c) $\overline{x}=1.0842$.

Answers to Review Problem Set #3: First Midterm

1. $y = 2\sin\left(\frac{\pi}{6}(x-1)\right)$, $\int_2^6 \left(2\sin\left(\frac{\pi}{6}(x-1)\right) - 1\right) dx = 12\sqrt{3}/\pi - 4 = 2.616$.
2. (a) $\int_0^1 \sqrt{13.0^2 + (-9.8t - 7.5)^2}\, dt$.
(b) $c = \epsilon a = 6$, so $b = \sqrt{a^2 - c^2} = 8$, $4\int_0^{\pi/2} \sqrt{100\sin^2\theta + 64\cos^2\theta}\, d\theta$.
(c) $\int_0^2 \left(10 - \frac{10}{0.1t+1}\right) dt$.
3. $\int_0^3 \sqrt{\dot{x}(t)^2 + \dot{y}^2}\, dt \approx 65.98$ meters (in practical terms 66 meters, since your data are given to only two places of accuracy).
4. (a) $x = 3t$, $y = 2 - 2t$. (b) $\sin\theta$ in x-direction, $\cos\theta$ in y-direction.
(c) $\theta = \pi t/2$. (d) $x = 3t + \sin(\pi t/2)$, $y = 2 - 2t + \cos(\pi t/2)$.

Answers to Review Problem Set #4: First Midterm

1. $\int_0^1 \cos(\pi x/2)dx - \int_0^1 (1-x)dx = (2/\pi) - (1/2) = 0.13662$.
2. (a) $\int_0^{0.7} \sqrt{6.86^2 + (-9.8t + 6.86)^2}\, dt$. (b) $\pi\int_0^2 9(1 - y^2/4)dy$.
3. $\overline{x} = \int_0^{\pi/2} x\sin x\, dx \Big/ \int_0^{\pi/2} \sin x\, dx$, bottom$=1$,
top$\approx \frac{\pi}{12}\left(\frac{\pi}{12}\sin\frac{\pi}{12} + \frac{\pi}{6}\sin\frac{\pi}{6} + \frac{\pi}{4}\sin\frac{\pi}{4} + \frac{\pi}{3}\sin\frac{\pi}{3} + \frac{5\pi}{12}\sin\frac{5\pi}{12} + \frac{1}{2}\cdot\frac{\pi}{2}\right) = 1.00573$, so $\overline{x} = 1.00573$. (Note: Later, when we come to integration by parts, we will be able to compute the top integral exactly, and find that it is actually equal to 1.)
4. (a) linear distance car traveled $= 3t^2 + 20t$, angular distance $\theta = 15t^2 + 100t$ radians. (b) About $20°$ beyond the top point (clockwise). (c) $dx/dt = (6t+20)(1 - \cos(15t^2 + 100t)) = 44.661$ when $t = 0.5$, $dy/dt = (6t+20)\sin(15t^2 + 100t) = -7.734$ when $t = 0.5$, so speed$= \sqrt{44.661^2 + 7.734^2} = 45.33$ m/sec.

Answers to Review Problem Set #5: First Midterm

1. Note that $\overline{y} = \overline{x}$ because of symmetry; the bottom integral in $\overline{x}$ is $\int_0^1 (0.5x+1)dx + \int_1^2 (0.5x + 1 - (2x-2))dx = 5/4 + 3/4 = 2$; the top integral in $\overline{x}$ is $\int_0^1 x(0.5x+1)dx + \int_1^2 x(0.5x + 1 - (2x-2))dx = \int_0^1 (0.5x^2 + x)dx + \int_1^2 (-1.5x^2 + 3x)dx = 5/3$, so that $\overline{x} = (5/3)/2 = 5/6$.
2. (a) $c = \sqrt{a^2 - b^2} = 24$, foci at $(-24, 0)$ and $(24, 0)$. (b) $e = c/a = 24/25 = 0.96$. (c) $x^2/25^2 + y^2/7^2 = 1$, so $x^2 = 625(1 - y^2/49)$, which is $625 \cdot 33/49$ when $y = -4$; $\frac{dh}{dt} = \frac{dV/dt}{dV/dh} = \frac{0.7}{\pi \cdot 625 \cdot 33/49} = 0.00052936$ m/sec.
3. (a) $x = 6t - 0.3\sin(15t)$, $y = 0.4 - 0.3\cos(15t)$. (b) When $t = 0.4$, $x = 2.4838$ and $y = 0.1119$, which is below and slightly to the right of the center, whose coordinates at that time are $(2.4, 0.4)$. (c) $dx/dt = 1.6792$, $dy/dt = -1.2573$, speed$= 2.098$ m/sec directed $36.8°$ below the horizontal.
4. (a) $v_{\text{horiz}}(t) = \frac{1}{2}t^2$, $v_{\text{vert}}(t) = \frac{6}{\pi}\sin(\pi t/6)$. (b) $\int_0^3 \sqrt{\left(0.5t^2\right)^2 + \left(\frac{6}{\pi}\sin(\pi t/6)\right)^2}\, dt$.
(c) $\frac{1}{2}\left(\frac{1}{2}f(0) + f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + \frac{1}{2}f(3)\right)$, where
$f(t) = \sqrt{0.25t^4 + (36/\pi^2)\sin^2(\pi t/6)}$ (final answer $= 6.007$).

Answers to Review Problem Set #1: Second Midterm

1. If you take 2 as the hypotenuse of the triangle and x as the leg opposite θ, then $x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$, $\sqrt{4 - x^2} = 2\cos\theta$, and the limits of integration become 0 to $\pi/6$. The top integral in $\overline{x}$ becomes: $\int_0^{\pi/6} 8\sin^3\theta\, d\theta = 8\int_0^{\pi/6} (1 - \cos^2\theta)\sin\theta\, d\theta = 8\int_1^{\sqrt{3}/2} (1 - u^2)(-du)$ (where $u = \cos\theta$). This leads to the value $\frac{16}{3} - 3\sqrt{3} = 0.13718$ for the top integral. The bottom integral is: $\int_0^{\pi/6} 4\sin^2\theta\, d\theta = 4\int_0^{\pi/6}\left(\frac{1 - \cos(2\theta)}{2}\right) d\theta = \left(2\theta - \sin(2\theta)\right)\Big|_0^{\pi/6} = \frac{\pi}{3} - \frac{\sqrt{3}}{2} = 0.18117$. So $\overline{x} = 0.13718/0.18117 = 0.75718$.

2. Since $x^4/(4 - x^2) = -x^2 - 4 + 16/(4 - x^2) = -x^2 - 4 - \frac{16}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$, we have:

Vol $= \pi \int_0^1 \left(-x^2 - 4 - \frac{16}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)\right) dx = \pi \left(-\frac{x^3}{3} - 4x - 4\ln\left|\frac{x-2}{x+2}\right|\right)\Big|_0^1 = \pi\left(-\frac{13}{3} + 4\ln 3\right) =$ 0.1920.

3. Vol$= f(x) = \pi(e^{-x})^2 x = \pi x e^{-2x}$. Set $0 = f'(x) = \pi\left(-2xe^{-2x} + e^{-2x}\right) = \pi e^{-2x}(-2x+1)$, so $-2x+1 = 0$, i.e., $x = 1/2$. So the maximum volume is $\pi \cdot \frac{1}{2} \cdot e^{-1} = \pi/2e = 0.57786$.

4. (a) $(x_A, y_A) = (8t, 40 - 4.9t^2)$, (b) $10 - 2t$, (c) $\arctan\big((40-4.9t^2)/8t\big) - \arcsin\big((10-2t)/20\big)$, (d)

$$\frac{-8t(9.8t) - 8(40-4.9t^2)}{(8t)^2 + (40-4.9t^2)^2} + \frac{0.1}{\sqrt{1-(0.5-0.1t)^2}}.$$

Answers to Review Problem Set #2: Second Midterm

1. (a) $f(1) = 0$ and $\lim_{x\to 0} f(x) = 0$. (b) $f'(x) = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} = 0$ gives $\ln x = -2$, so $x = 1/e^2$ and the point is $(1/e^2, -2/e)$. (c) The top integral is

$$\int_0^1 x^{3/2}\ln x\, dx = \frac{2}{5}x^{5/2}\ln x\,\Big|_0^1 - \int_0^1 \frac{2}{5}x^{5/2}\cdot\frac{1}{x}\,dx = -\frac{2}{5}\int_0^1 x^{3/2}dx = -\frac{4}{25}.$$

Similarly, the bottom integral is $\int_0^1 x^{1/2}\ln x\,dx = -\frac{4}{9}$. So $\overline{x} = 9/25 = 0.36$.

2. (a) Write $1/(x^2+5x+4) = \frac{1}{-1-(-4)}\left(\frac{1}{x+1} - \frac{1}{x+4}\right)$, so we get $\frac{1}{3}\ln\left(\frac{x+1}{x+4}\right)\Big|_0^\infty = \frac{1}{3}\ln 4 = 0.4621$. (b) Write $x^2 + 2x = (x+1)^2 - 1^2$; if you take $x+1$ for the hypotenuse of the triangle and 1 for the leg adjacent to θ, then you have $x + 1 = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$, $\sqrt{x^2+2x} = \tan\theta$, and the limits of integration become 0 to $\pi/2$ (because the triangle has $\theta = 0$ when $x = 0$ and has angle θ approaching $\pi/2$ as $x \longrightarrow \infty$). In the integral everything cancels, and you get simply $\int_0^{\pi/2} d\theta = \pi/2 = 1.5708$.

3. $\pi\int_0^{\pi/2} x\cos^2 x\,dx = \pi\int_0^{\pi/2} x\frac{1+\cos(2x)}{2}dx = \pi\left(\frac{\pi^2}{16} + \frac{1}{2}\int_0^{\pi/2} x\cos(2x)dx\right) =$
$= \frac{\pi^3}{16} + \frac{\pi}{2}\left(\frac{1}{2}x\sin(2x)\,\Big|_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2}\sin(2x)dx\right) = \frac{\pi^3}{16} - \frac{\pi}{4} = 1.1525.$

4. $y_A(t) = 5e^{0.03t}$, $y_B(t) = 15e^{-0.05t}$, so minimize $f(t) = 5e^{0.03t} + 15e^{-0.05t}$. Set $0 = f'(t) = 0.15e^{0.03t} - 0.75e^{-0.05t}$, so $0.15e^{0.03t} = 0.75e^{-0.05t}$, and $e^{0.03t}/e^{-0.05t} = 0.75/0.15$, i.e., $e^{0.08t} = 5$. Take ln of both sides and divide by 0.08 to get $t = (\ln 5)/0.08 = 20.118$ hours.

Answers to Review Problem Set #3: Second Midterm

1. (a) $\int_0^1 (1+2y)\arctan(y)dy = (y+y^2)\arctan(y)\,\Big|_0^1 - \int_0^1 \frac{y+y^2}{1+y^2}dy = \frac{\pi}{2} - \int_0^1\left(1 + \frac{y}{1+y^2} - \frac{1}{1+y^2}\right)dy$
$= \frac{\pi}{2} - \left(y + \frac{1}{2}\ln(1+y^2) - \arctan\, y\right)\Big|_0^1 = \frac{3}{4}\pi - 1 - \frac{1}{2}\ln 2.$

(b) Write $x - x^2 = 0.5^2 - (x-0.5)^2$, take 0.5 as the hypotenuse and $x - 0.5$ as the leg opposite θ; then $\int\frac{x\,dx}{\sqrt{x-x^2}} = \int\frac{0.5(\sin\theta+1)0.5\cos\theta\,d\theta}{0.5\cos\theta} = -0.5\cos\theta + 0.5\theta + C = -\sqrt{x-x^2} + \frac{1}{2}\arcsin(2x-1) + C.$

(c) $\int_\pi^{4\pi}\cos^3(t/4)dt = \int_\pi^{4\pi}\left(1-\sin^2(t/4)\right)\cos(t/4)dt = \int_{\sqrt{2}/2}^0 (1-u^2)(4\,du)$ (where $u = \sin(t/4)$); this is computed to be $-5\sqrt{2}/3$.

2. $\ln y = \ln\left((\ln x)^{e^x}\right) = e^x\ln(\ln x)$, which leads to $\frac{1}{y}\frac{dy}{dx} = e^x\left(\frac{1}{x\ln x} + \ln(\ln x)\right)$, and then $\frac{dy}{dx} = (\ln x)^{e^x}\left(\frac{1}{x\ln x} + \ln(\ln x)\right)$.

3. (a) $dv/dt = -kv$; (b) $v(t) = 5e^{-kt}$; (c) $s(t) = \int_0^t 5e^{-ku}du = \frac{5}{k}\left(1-e^{-kt}\right)$; (d) (1) $5e^{-0.8} = 2.247$ m/sec, (2) $50(1-e^{-0.8}) = 27.53$ m, (3) $s/r = 137.67$ rad, (4) $v/r = 11.23$ rad/sec; (e)

$$\frac{dL}{dt} = \frac{dL}{d\theta}\frac{d\theta}{dt} = \left(\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\right)\cdot 11.23 = 11.23\sqrt{r^2(1-\cos\theta)^2 + r^2\sin^2\theta} = 1.247\,\frac{\text{m}}{\text{sec}}$$

when $r = 0.2$, $\theta = 137.67$ rad.

Answers to Review Problem Set #4: Second Midterm

1. (a) $\int_{10\sqrt{3}}^{\infty} \frac{1}{100} \frac{dx}{1+(x/10)^2} = \frac{1}{10} \int_{\sqrt{3}}^{\infty} \frac{du}{1+u^2} = \frac{1}{10}$ arctan $u \Big|_{\sqrt{3}}^{\infty} = \frac{1}{10}\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = \pi/60$.
(b) $\int (x+2)\left(\frac{1}{x-2} - \frac{1}{x-1}\right) dx = \int \frac{x+2}{x-2} dx - \int \frac{x+2}{x-1} dx = \int \left(1 + \frac{4}{x-2}\right) dx - \int \left(1 + \frac{3}{x-1}\right) dx = 4 \ln|x - 2| - 3 \ln|x-1| + C$. (c) Set $u = 2(x+1)$, so that:
$\int_{-1/2}^{1/2} x\, e^{2(x+1)} dx = \frac{1}{2} \int_1^3 (\frac{1}{2}u - 1) e^u du = \frac{1}{4} \int_1^3 u e^u du - \frac{1}{2} \int_1^3 e^u du = \frac{1}{4}\left(ue^u|_1^3 - e^u|_1^3\right) - \frac{1}{2} e^u|_1^3 = \frac{1}{2}e$.
(d) $\int_0^{\pi} \sin^5(x/3) dx = \int_0^{\pi} \left(1 - \cos^2(\frac{x}{3})\right)^2 \sin(\frac{x}{3}) dx = \int_1^{1/2} (1-u^2)^2(-3\, du)$ (where $u = \cos(\frac{x}{3})$). This gives $3(-\frac{1}{5}u^5 + \frac{2}{3}u^3 - u)|_1^{1/2} = 0.331$.
2. (a) $\arctan\Big((7 - t - 2\sin(2\pi t))\ /\ (3 + 1.4t - 2\cos(2\pi t))\Big)$. (b) Let $y(t)$ be the top function and $x(t)$ the bottom function in the Arctan. Then $\frac{d}{dt}$ arctan $\left(\frac{y}{x}\right) = \frac{1}{1+(y/x)^2} \cdot \frac{x\dot{y} - y\dot{x}}{x^2} = \frac{x\dot{y} - y\dot{x}}{x^2+y^2}$; multiply by $180/\pi$ to convert to degrees; the result is:

$$\frac{180}{\pi} \frac{(3 + 1.4t - 2\cos(2\pi t))(-1 - 4\pi\cos(2\pi t)) - (7 - t - 2\sin(2\pi t))(1.4 + 4\pi\sin(2\pi t))}{(3 + 1.4t - 2\cos(2\pi t))^2 + (7 - t - 2\sin(2\pi t))^2}.$$

3. (a) $0 = f'(x) = (1/2\sqrt{x}) - (1/x)$ gives $x = 4$, i.e., the point $(4, 0.6137)$. (b) $f''(x) = -\frac{1}{4}x^{-3/2} + x^{-2}$, which is positive if $x = 4$, so that the point in part (a) is a minimum. (c) $f''(x) = 0$ when $x = 16$, and the point $(16, 1.2274)$ is an inflection; the curve is concave up for $x < 16$ and concave down for $x > 16$. (d) $f(x)$ approaches $+\infty$ as $x \longrightarrow 0$ and as $x \longrightarrow \infty$.
(e)

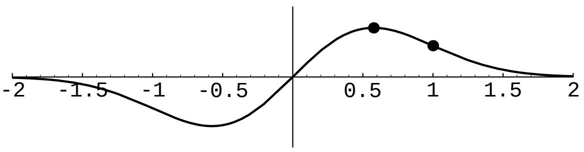
Answers to Review Problem Set #1: Final Exam

1. (a) $0 = f'(x) = \frac{1}{x} - \frac{2.5 \cdot 0.5}{1+(0.5x)^2}$, so that $1/x = 1.25/(1 + 0.25x^2)$, i.e., $1 + 0.25x^2 - 1.25x = 0$, or $x^2 - 5x + 4 = 0$, which has roots $x = 1$ and $x = 4$; hence, the critical points are $(1, -1.159)$ and $(4, -1.382)$; by the 2nd derivative test, the first is a max and the second is a min; (b) $\ln x$; (c) $\ln x - (5\pi/4) = \ln x - 3.927$.
2. (a) $\pi \int_0^{12} g(y)^2 dy \approx 2\pi(\frac{1}{2} \cdot 5.4756 + 6.9169 + 8.4100 + 9.9856 + 11.5600 + 13.1769 + \frac{1}{2} \cdot 14.8225) = 378\, \text{cm}^3$; (b) $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = \pi(3.85)^2 \cdot 0.05 = 2.33\, \text{cm}^3/\text{sec}$.
3. (a) $K\Delta x/(r^2+x^2)$, (i) $Kr\Delta x/(r^2+x^2)^{3/2}$, (ii) $Kx\Delta x/(r^2+x^2)^{3/2}$; (b) in the direction perpendicular to wire: $\int_0^{\infty} \frac{Kr\, dx}{(r^2+x^2)^{3/2}}$, use inverse trig substitution with $x = r\tan\theta$, $dx = r\sec^2\theta\, d\theta$, $\sqrt{r^2+x^2} = r\sec\theta$, to transform the integral to $\int_0^{\pi/2} \frac{Kr^2 \sec^2\theta\, d\theta}{r^3\sec^3\theta} = \frac{K}{r}\int_0^{\pi/2}(1/\sec\theta) d\theta = \frac{K}{r}\int_0^{\pi/2} \cos\theta\, d\theta = \frac{K}{r}$; in the direction parallel to wire: $\int_0^{\infty} \frac{Kx\, dx}{(r^2+x^2)^{3/2}}$, use u-substitution $u(x) = r^2 + x^2$ to get: $\int_{r^2}^{\infty} \frac{1}{2} K u^{-3/2} du = -Ku^{-1/2}\Big|_{r^2}^{\infty} = \frac{K}{r}$; (c) the horizontal and vertical components of the force are the same (K/r in both cases), so the force points at a 45° angle.
4. (a) $\frac{dv}{dt} = g - 0.07v$; (b) $v(t) = -140\left(1 - e^{-0.07t}\right)$; (c) 41.34 m/sec downward;
(d) $\frac{dv}{dt} = g - 2.0v$, $v(t) = -4.9 - 36.44e^{-2t}$; (e) 1.75 sec.
5. (a) $2 - 0.2t$; (b) $10 - 0.1t^2$, empty when $t = 10$ min; (c) $\frac{dy}{dt} = -\frac{2}{10-0.1t^2}y + 0.56(1 - 0.1t)$.

Answers to Review Problem Set #2: Final Exam

1. (a) $0 = f'(x) = (-3x^2+1)e^{-1.5x^2}$, so $x = \pm 1/\sqrt{3}$, and the critical points are $(0.57735, 0.35018)$ and $(-0.57735, -0.35018)$; since $f''(x) = (-3x(-3x^2+1) - 6x)e^{-1.5x^2} = 9x(x^2-1)e^{-1.5x^2}$, the second derivative test shows that the first of these points is a max, the second is a min; (b) inflections at

$(0,0)$, $(1, 0.22313)$ and $(-1,-0.22313)$; (c) $f(x)$ rapidly approaches the x-axis; (d) (see the figure below).



2. (a) $\theta = \arctan(a/b)$; (b) $\frac{d\theta}{dt} = \frac{180}{\pi}(b\dot{a} - a\dot{b})/(a^2+b^2)$.
3. $\int_0^8 \sqrt{\dot{x}^2+\dot{y}^2}dt \approx 2\Big(\frac{1}{2}\sqrt{10.0^2+0^2}+\sqrt{9.8^2+8.0^2}+\sqrt{9.4^2+15.2^2}+\sqrt{9.0^2+22.0^2}+\frac{1}{2}\sqrt{8.4^2+28.2^2}\Big) =$ 148 meters.
4. (a) $\frac{dy}{dt} = -\frac{r}{V}y$; (b) $0.01y_0 = y_0 e^{-(r/V)T}$, so $T = -(V/r)\ln(0.01) = V\ln 100/r = 4.605V/r$.
5. (a) $dy/dt = 0.12y - 1500$; (b) $y = 12500 - 4500e^{0.12t}$; (c) 8.51 years; (d) 12770 rhinos; (e) if you buy an \$8000 car, no money down, 12% interest on the loan, making monthly payments of \$125, how long does it take you to pay off the car loan, and how much do you end up having paid for the car in all?
6. (a) $\frac{dv}{dt} = -8 - 0.05(v+5t)$; (b) $\frac{dv}{dt} = -0.05v - (8+0.25t)$.

Answers to Review Problem Set #3: Final Exam

1. (a) $x = 17.32t$, $y = -4.9t^2 - 10t + 50$; (b) $\theta = \arctan\Big(\big(-4.9t^2 - 10t + 50\big)/17.32t\Big)$;
(c) $\frac{d\theta}{dt} = \Big(17.32t(-9.8t-10) - 17.32(-4.9t^2-10t+50)\Big)/\Big(300t^2 + (-4.9t^2-10t+50)^2\Big)$; when $t = 2$ this gives -0.9215 rad/sec.
2. (a) $v(t) = 55 + \int_0^t \left(24/\sqrt{1-u^2}\right) du = 24\arcsin(t) + 55$; (b) $v(\frac{1}{2}) = 67.57$ miles/hr;
(c) $s(\frac{1}{2}) - s(0) = \int_0^{1/2} v(t)dt = \int_0^{1/2} 24\arcsin(t) + 55\, dt =$
$= 24\left(t\arcsin(t)\,\Big|_0^{1/2} - \int_0^{1/2} t(1-t^2)^{-1/2}dt\right) + 55t\,\Big|_0^{1/2} =$
$= 24\cdot\frac{1}{2}\cdot\frac{\pi}{6} - 24\left(-\sqrt{1-t^2}\,\Big|_0^{1/2}\right) + \frac{1}{2}\cdot 55 = 2\pi + 12\sqrt{3} + 3.5 = 30.57$ miles.
3. (a) $y = (1/x) + \ln x$; $y' = (-1/x^2) + (1/x) = 0$ gives the point $x = 1$, $y = 1$ (a min pt);
$y'' = (2/x^3) - (1/x^2) = 0$ gives the point $x = 2$, $y = 0.5 + \ln 2 = 1.193$;
(b) approaches $+\infty$; (c) approaches $+\infty$; (e) $\int_{1/2}^1 \sqrt{1 + (-(1/x^2)+(1/x))^2}\, dx \approx$
$\approx \frac{1}{8}\left(\frac{1}{2}f(\frac{1}{2}) + f(\frac{5}{8}) + f(\frac{3}{4}) + f(\frac{7}{8}) + \frac{1}{2}f(1)\right) = \frac{1}{8}(1.1180 + 1.3862 + 1.0943 + 1.0132 + 0.5) = 0.6390.$
4. (a) $\frac{dy}{dt} = -\frac{r}{V}(y - kV) = -(0.6 + 0.4\sin(\frac{\pi t}{10}))(y - 5k)$; (b) separation of variables leads to $y - 5k = Ce^{-0.6t + (4/\pi)\cos(\pi t/10)}$, where $C = (y_0 - 5k)e^{-4/\pi}$, i.e., $y(t) = 5k + (y_0 - 5k)e^{-0.6t - (4/\pi)(1-\cos(\pi t/10))}$;
(c) increases if $y_0 < 5k$, decreases if $y_0 > 5k$.
5. $dy/dt = 0.025y + (1.0 + 0.1t)$.

Answers to Review Problem Set #4: Final Exam

1. (a) $\theta = \arctan(\frac{5}{y})$; (b) $\frac{d\theta}{dt} = \frac{d\theta}{dy}\cdot\frac{dy}{dt} = 1/\left(1 + \frac{25}{y^2}\right)\cdot\frac{-5}{y^2}\cdot 0.5 = -\frac{2.5}{y^2+25} = -\frac{1}{260}$ rad/min $= -0.22$ deg/min.
2. $f'(x) = x^x(\ln x + 1)$; critical point at $(1/e, 0.692)$; $f''(x) = x^x\left(\frac{1}{x} + (\ln x + 1)^2\right)$, which equals $0.692e$ at the critical point — so the critical point is a minimum.
3. $\Delta x\Big(\frac{1}{2}\sqrt{1+f'(0)^2} + \sqrt{1+f'(0.5)^2} + \sqrt{1+f'(1)^2} + \sqrt{1+f'(1.5)^2} + \sqrt{1+f'(2)^2} +$
$+\sqrt{1+f'(2.5)^2} + \frac{1}{2}\sqrt{1+f'(3)^2}\Big) = \frac{1}{2}(0.7071 + 1.3521 + 1.2440 + 1.1461 + 1.0808 +$
$+1.0218 + 0.5056) = 3.529.$
4. position of shadow$= u = x + y = 2\theta + 2 - \sin\theta - \cos\theta$; $\frac{d\theta}{dt} = \frac{1}{r}\cdot$speed $= 2.5$ rad/sec; $\frac{du}{dt} = \frac{du}{d\theta}\cdot\frac{d\theta}{dt} =$ $(2 - \cos\theta + \sin\theta)\cdot 2.5$; for $\theta = 2\pi/3$ this gives 8.415 m/sec.

5. (a) $\frac{dy}{dt} = ky - a$; (b) $y(t) = \frac{a}{k} - (\frac{a}{k} - y_0)e^{kt}$; (c) $a = ky_0/(1 - e^{-kT})$. (d) Note that $ky_0 = a(1 - e^{-kT})$. To find when $ky(t) = \frac{1}{2}a$, we set $\frac{1}{2}a = ky = a - (a - ky_0)e^{kt}$. This gives $(a - ky_0)e^{kt} = a - \frac{1}{2}a = \frac{1}{2}a$. Replacing ky_0 by $a(1 - e^{-kT})$ and dividing through by a, we get $(1 - (1 - e^{-kT}))e^{kt} = 1/2$, i.e., $e^{k(t-T)} = 1/2$. Taking ln of both sides and dividing through by k, we finally get $t = T - \frac{\ln 2}{k}$. In this problem, $T = 20$ years and $k = 10\% = 0.1$, so that $t = 20 - 10 \ln 2 = 13.07$ years.

6. $\frac{dy}{dt} = -k_1 y + I_0 e^{k_2 t}$. 7. time units are months, so convert 12% annual interest to 1% monthly interest; $\frac{dy}{dt} = 0.01y - \alpha(1 - 0.01t)$.