Answers

Answers to Selected Homework 1 Problems. 1. (a) $8\sqrt{2}/15 = 0.754$, (b) $1/12$, (c) $(n-1)/(n+1)$, (d) 4.5 (the curves intersect at $(-3,2)$ and $(0,-1)$), (e) $4/3$, (f) $(2/3)-(2/\pi) = 0.030$, (g) $(3/\pi)(1-0.5\sqrt{3}) - (1/8) = 0.00294$.
2. (a) $\sqrt{2}-1 = 0.414$, (b) $46/3$.
3. (a) net$=1/\pi$, total$=5/\pi$, (b) net$=0$, total$=245$, (c) net$=20$, total$=28$, (d) net$=(1.5/\pi) - (1/2) = -0.0225$, total$=0.25 + (4.5-3\sqrt{3})/\pi = 0.0284$.
4. (a) $a_x(t) = (F/m)\cos(\omega t)$, $a_y(t) = (F/m)\sin(\omega t)$, (b) $v_x(t) = (F/m\omega)\sin(\omega t)$, $v_y(t) = (F/m\omega)(1-\cos(\omega t))$, (c) $x(t) = \left(F/m\omega^2\right)(1-\cos(\omega t))$, $y(t) = \left(F/m\omega^2\right)(\omega t - \sin(\omega t))$, (d) a cycloid running up the y-axis (rather than the x-axis), (e) a circle if $v_0 = -F/m\omega$.

Answers to Selected Homework 2 Problems. 1. hits ground at (a) 3 sec, (b) 4.5 sec, (c) 2 sec. 2. 44.3 m/sec, 0.232 sec.
3. $v_0 = -38$ ft/sec, $s_0 = 590$ ft. 4. 1032 ft.
5. (a) $8\pi/3$, (b) $\pi/30$, (c) $0.5\pi^2 - \pi = 1.793$. 6. (a) 22.8π, (b) 14.8π, (c) 20π, (d) 4π.
7. (a) $\frac{4}{3}\pi ab^2$, (b) $\frac{4}{3}\pi a^2 b$. 8. $\pi h^2\left(r - \frac{1}{3}h\right)$. 9. $\int_0^H \frac{\sqrt{3}}{4}(\frac{x}{H}s)^2\,dx = \frac{\sqrt{3}}{12}s^2H$.

Answers to Selected Homework 3 Problems. 1. (a) 6 min 17 sec, (b) 0.03183 units/min.
2. (a) 20 min 57 sec, (b) 0.03183 units/min. 3. 0.6283 ft^3/min. 4. $dV/dt = 0.04\pi\left(4-(-2+0.04t)^2\right)$.
5. $dV/dt = 0.09\pi\left(4-(-2+0.04t)^2\right)$. 6. 1.21 cm/sec. 7. cross-sectional area $= \frac{1}{4}\sqrt{3}(\frac{h}{H}s)^2$.

Answers to Selected Homework 4 Problems. 1. (a) $x^2/4+4y^2 = 1$, (b) $x^2/100+y^2/64 = 1$, (c) $x^2/625 + y^2/576 = 1$, (d) $x^2 + y^2/9 = 1$, (e) $x^2/16 + y^2/25 = 1$.
2. (a) $(\pm 3, 0)$, $e = 0.6$; (b) $(0, \pm 24)$, $e = 0.96$; (c) $(\pm 0.3727, 0)$, $e = 0.7454$. 3. 10.
4. (a) $\int_0^b \sqrt{1+n^2x^{2(n-1)}}\,dx$; (b) $\frac{8}{27}\left((1+9b/4)^{3/2}-1\right)$. 5. (a) $\int_0^4 \sqrt{98 + (-9.8t+7\sqrt{2})^2}\,dt$; (b) $\int_0^{\pi/2} \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}\,d\theta$; (c) r$\int_0^{2\pi} \sqrt{1.25-\cos\theta}\,d\theta$. 6. (a) $1/\sqrt{7} = 0.3780$; (b) $\frac{d}{dt}(x + 2.5y) = 0.3780(1 + 2.5\sqrt{3}/2) = 1.1964$; (c) $u = x + y\cot\alpha$, so $\frac{du}{dt} = \frac{dx}{dt} + \frac{dy}{dt}\cot\alpha + y(-\csc^2\alpha)\frac{d\alpha}{dt} = 1.1964 - y(\csc^2\alpha)\left(-15\cdot\frac{\pi}{180}\right) = 1.1964 + 0.5\cdot 7.2509\cdot\pi/12 = 2.1455$ units/hour.

Answers to Selected Homework 5 Problems. 1. 55.4 m/sec 2. (a) 0.790766, (b) 1.183465, (c) 0.987116, (d) 0.997943. 3. 8.878.
4. (a) 0.58726, (b) 0.58622 (limits of integration are $\pi/3$ and $\pi/2$).
5. (b) 0.2402 m, (c) 0.4541 m.
6. (a) $x(8) - x(0) = \int_0^8 \dot{x}(t)dt \approx 34.45$, $y(8) - y(0) = \int_0^8 \dot{y}(t)dt \approx 46.35$, and finally $\sqrt{34.45^2 + 46.35^2} = 57.75$; (b) $\int_0^8 \sqrt{(dx/dt)^2 + (dy/dt)^2}dt \approx 59.29$.

Answers to Selected Homework 6 Problems. 1. (a) (2,1), (b) (1.5,1.2), (c) $(0.4244r, 0)$, (d) $(0, -0.8377)$.
2. at 39.37° northern latitude & 116.6° western longitude (right between Summit Mountain and Hickison Summit on the border between Lander and Eureka counties). 3. 0.5466.

Answers to Selected Homework 7 Problems. 1. (a) $2x^2/(4+x^2) + 2x \arctan(x/2)$.
(b) $\Big(\arcsin(x^2) - 2x(x+1)(1-x^4)^{-1/2}\Big) \Big/ \Big(\arcsin(x^2)\Big)^2$.
(c) $-1/2\sqrt{x-x^2}$, (d) $\arcsin x$. 2. (a) $0.25 \arcsin(0.4x) + C$, (b) $0.5\sqrt{2} \arctan(\sqrt{2}\,x) + C$.
3. 31.32°. 4. $\pi^2/4$. 5. (0.2559,0).
6. (a) max: $(1, 0.2854)$, min: $(-1, -0.2854)$, (b) max: $(-\sqrt{3}, 0.685)$, min: $(\sqrt{3}, -0.685)$.
7. (a) $\pi - (2/3)$, (b) $2\sqrt{3} - 2\pi/3$. 8. 4.584 deg/sec.
9. (a) $\arccos\big((a^2+b^2-x^2)/2ab\big)$, (b) $2c \,/\, \sqrt{(2ab)^2 - (a^2+b^2-c^2)^2}$.

Answers to Selected Homework 8 Problems. 1. (a) $1/x$, (b) $(4\ln^3 x)/x$, (c) $(2\ln(\ln x))/(x \ln x)$, (d) $1/(\sin x \cos x)$, (e) $-\arctan(x/4)$.
2. (a) both derivatives are 0; (b) use the fact that only a constant function has derivative zero, find the constant by setting $x = 1$, and then replace x by b in the first function and by a in the second function.
3. (a) $\ln 3$, (b) $\frac{3}{2}\ln 2$, (c) $\frac{2}{3}\ln 3$, (d) $\frac{3}{2}\ln 3 - \ln 2$, (e) $\ln \pi$, (f) $\frac{1}{2}\ln(r+x)$.
4. (a) $-\ln 2$, (b) $\ln \sin x + C$, (c) $\ln(\ln x) + C$, (d) $(1/\ln 2) - (1/\ln 3) = 0.5325$, (e) $2 \ln 3$.
5. (a) 1.4427, (b) 1.3863, (c) 0.4413. 6. (a) 0.4449, (b) 0.09436. 7. 4.3552.
8. 11.18, 14.14, 22.36.
9. (a) 0, (b) 0, (c) 0, (d) 0. 10. (a) $(1 + \ln x)x^x$, (b) $\big(\sqrt{x}\cot x + (\ln \sin x)/2\sqrt{x}\big)(\sin x)^{\sqrt{x}}$, (c) $\big(\frac{1}{\ln x} + \ln(\ln x)\big)(\ln x)^x$, (d) $\big(\frac{2}{x}\ln x\big)\, x^{\ln x}$, (e) $\Big(-(\arctan x)\tan x + \frac{\ln \cos x}{1+x^2}\Big)\,(\cos x)^{\arctan x}$.
11. Let a be the number of hours before noon when it started snowing, and let $s(t)$ be the distance traveled after t hours of plowing. Show that ds/dt is inversely proportional to $t+a$, and then use the method in section 2 of Chapter 1 to find a formula for $s(t)$; you should find that $s(t) = C\ln((t+a)/a)$. (The value of the constant C makes no difference.) You're given that $s(2) = \frac{3}{2}\,s(1)$. Use the properties of ln and some algebra to get a quadratic equation for a, from which you find that $a = (-1+\sqrt{5})/2$ (this number is called the "golden ratio"). Snow started falling at 11:23 a.m.

Answers to Selected Homework 9 Problems. 1. (a) 2, (b) ∞, (c) ∞, (d) 2, (e) $\pi/2$, (f) π, (g) $\pi/3$, (h) $\pi/2$, (i) π, (j) 1, (k) ∞, (l) $\pi/4$.
2. $\pi n/(2-n)$, infinite volume if $n \geq 2$. 3. $(n-1)/(n-2)$.

Answers to Selected Homework 10 Problems. 1. (a) 1, (b) 0.9917. 2. (a) $3x$, (b) $-x^2$, (c) x/y^2, (d) x, (e) $\frac{1}{2}x^2$. 3. (a) $20 \ln x$, (b) $x^2 + x$,
(c) $\arcsin e^x$, (d) $\pm\sqrt{2\ln x}$, (e) $1/(1+e^{0.01x})$. 4. (a) $e^{2x}\left(\frac{1}{1+x^2} + 2\arctan x\right)$,
(b) $e^{-\sin(\pi x)}\big(1 - \pi x \cos(\pi x)\big)$, (c) e^{x+e^x}, (d) $e^{-\cos x}(2x + x^2 \sin x)$, (e) $e^{\arcsin x}/\sqrt{1-x^2}$.
5. (a) $\frac{d}{dx}\cosh x = \sinh x$, $\frac{d}{dx}\sinh x = \cosh x$, (b) 1.
6. (a) 3.6, (b) $\frac{1}{2} - \frac{1}{2e}$, (c) $e - \frac{1}{e}$, (d) $1/\ln 10$. 7. $\frac{15\pi}{16\ln 4} \approx 2.1245$.
8. (a) max at $(\pi/6, 1.9477)$, inflection points at $(\pi/18, 1.3956)$ and $(5\pi/18, 1.3956)$.
(b) max at $(0.6065, 0.1839)$, inflection at $(0.2231, 0.0747)$.

Answers to Selected Homework 11 Problems. 1. \$4932. 2. (a) 11.55 yrs, (b) 6.93 yrs, (c) 3.85 yrs; (a) 11.90 years, (b) 7.27 yrs, (c) 4.19 yrs. 3. 20480. 4. 1020201, 1491825, 34.66 yrs. 5. $250e^{0.007675(t-1650)}$, A.D. 2250. 6. 11.33%, 6.12 yrs. 7. 6.05 times 8. (a) \$4034300, (b) $100e^{0.13t}$, (c) $\int_0^{10} 52 \cdot 100e^{0.13t} = 40000e^{0.13t}|_0^{10} =$ \$106772, (d) \$44241000, (e) about

2/3 day. 9. $8033\ln(y_0/y_1)$. 10. 6.64 hrs. 11. $100e^{-0.0112t}$, 62.1 yrs. 12. \$11477. 13. (a) $v(t) = v_0e^{-kt}$; (b) $s(t) = \frac{v_0}{k}(1-e^{-kt})$; (c)$\arctan(ky_0/v_0)$.

Answers to Selected Homework 12 Problems. 1. 2.5587. 2. (a) $-\frac{1}{\pi}\sqrt{2+\cos(2\pi t)}+C$, (b) $-1/\left(7a\sin^7(ax+b)\right)+C$. 3. 2.0191.
4. (a) $4/(3\pi) = 0.4244$, (b) $3/8 = 0.375$, (c) $16/(15\pi) = 0.3395$, (d) $5/16 = 0.3125$, (e) $32/(35\pi) = 0.2910$.

Answers to Selected Homework 13 Problems. 1. (a) $-\frac{x}{a}\cos(ax+b)+\frac{1}{a^2}\sin(ax+b)+C$,
(b) $x^2\sin x + 2x\cos x - 2\sin x + C$,
(c) $x\arctan x - \frac{1}{2}\ln\left(1+x^2\right)+C$, (d) $\left(x^3-3x^2+6x-6\right)e^x+C$,
(e) $\frac{1}{2}x^2\arctan x - \frac{x}{2}+\frac{1}{2}\arctan x + C$.
2. (a) $\left(-\frac{1}{k},\frac{1}{4}\right)$, (b) $(9/16,0)$, (c) $\left(\left(\frac{n+1}{n+2}\right)^2,0\right)$, (d) 0.18484. 3. 0.60672.

Answers to Selected Homework 14 Problems. 1. (a) 0.2747, (b) 0.09861, (c) 0.05371, (d) 0.4621, (e) 0.06112, (f) 0.04251, (g) 0.2118, (h) 0.2, (i) 0.1903, (j) $-\frac{1}{5}\ln(6) = -0.35835$.
2. (a) (0.7095,0), (b) (0.7912,0). 3. $\frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}+C$.

Answers to Selected Homework 15 Problems. 1. $Arcsin(x/r)+C$. 2. (a) $\ln|x+\sqrt{x^2+r^2}|+C$, (b) $\ln|x+\sqrt{x^2-r^2}|+C$. 3. $(0.37616,0)$. 4. 0.93313. 5. 0.48121.

Answers to Selected Homework 16 Problems. 1. \$27.98, \$1007.30. 2. (a) \$379.15, \$27749, (b) 7.7 years, (c) \$2613.91 (1st year), \$1574.15 (5th year). 3. \$48633.80, \$9726.76.
4. (a) \$5613.14, (b) only \$17.01 more, (c) 22.57 years.
5. $T - (\ln 2)/k = 13$ years and 8.22 months.
6. (a) \$44754, (b) \$328.12, (c) 64.9 years, (d) \$298.36. 8. 9:43 p.m. 9. 7.27 hours
10. 4:58 a.m. 11. 8:57 a.m. 12. (a) 0.2, (c) 40.3 m/sec upward, (d) 1.71 sec.
13. $s(t) = \begin{cases} 98t + 980\left(e^{-0.1t}-1\right), & \text{for } 0\le t\le 10; \\ 7t - 39.25e^{-1.4(t-10)} + 329.77, & \text{for } t > 10, \end{cases}$ 24.3 sec.
14. $v(t) = \begin{cases} 9.8t, & \text{for } 0\le t\le 7; \\ 19.6 + 49.0e^{-0.5(t-7)}, & \text{for } 7\le t\le 15. \end{cases}$
15. (a) $y = 10-5e^{-0.03t}$, (b) $y = 10+10e^{-0.03t}$, (c) $y = 10$. 16. 7.42 lb.
17. (a) $y(t) = 48\left(1-e^{-t/12000}\right)$, (b) 36 min.
18. (c) $Vk + VP/r$, (e) Superior 431 yr, Michigan 71.4 yr, Erie 6.05 yr, Ontario 17.6 yr.
19. (a) $dy/dt = -ky+I$, (b) 339 million, 400 million. 20. $R = 80565$ per year; if $R = 300000$, then $y(t) = 3147600 - 147600e^{0.0953t}$, and the salmon will die out after 32.1 years.

Answers to Selected Homework 17 Problems. 1. (a) $y = \sqrt{3x/(6+x)}$, (b) $y = 1/(10-\frac{1}{3}x^3)$, (c) $y = \sqrt{\frac{2}{\pi}\sin(\pi x) + 1}$.
2. (a) $y(t) = M/\left(1+Ce^{-kt}\right)$, (b) $100/\left(1+19e^{-0.9474t}\right)$, 6.216 years.
3. $y_0(1-0.02t)^{2.5}$, domain is $0\le t\le 50$. 4. (a) $v_\infty = 49$ m/sec, (b) $dv/(100+v)(49-v) = 0.002dt$, then get $(100+v)/(49-v) = e^Ce^{0.298t}$, where $e^C = 100/49$ (since $v(0)=0$), finally end up with $v(t) = 49 - \left(73.01/\left(e^{0.298t}+0.49\right)\right)$, (c) $\int_0^3 49 - \left(73.01/\left(e^{0.298t}+0.49\right)\right)$.
(d) $ds/dv = v/\left(9.8-0.102v-0.002v^2\right)$, so $\int_0^{48} v\,dv/\left(9.8-0.102v-0.002v^2\right) = 508$ m.
5. 205 m. 6. (a) $\frac{dy}{dt} = -\frac{r(t)}{V}y$, (b) $y_0e^{-0.02t-0.0025t^2}$, (c) $y_0e^{-0.01t+\frac{0.01}{\pi}\cos(\pi(t-0.5))}$.
7. $dy/dt = -2y/(100+0.25t^2)$, $y(t) = y_0e^{-0.4\arctan(0.05t)}$, $y(\infty) = y_0e^{-0.2\pi} = 0.5335y_0$.
8. (a) $\frac{dy}{dt} = -\frac{r(t)}{V}(y-kV)$, where $V = 100$. (b) $y(t) = 100k + (y_0-100k)e^{-0.01\int_0^t r(u)du}$.
(c) $\int_0^{10} r(u)du = 26.73$; $e^{-0.01\cdot 26.74} = 0.765$, i.e., 23.5% decrease.

(d) $y(10) = 40 + (30-40)e^{-0.01\cdot 26.74} = 32.35$ gm.
9. (a) $\frac{dv}{dt} = -8 - 0.4v - 0.01v^2$, (b) $v(t) = 20\big(\tan(1.19 - 0.2t) - 1\big)$, (c) 2.024 sec.
10. (a) $(m-\alpha t)\frac{dv}{dt} = F - Kv$, (b) $v(t) = \frac{F}{K} + \big(v_0 - \frac{F}{K}\big)\,\big(1 - \frac{\alpha t}{m}\big)^{K/\alpha}$.
11. $y(t) = y_0\sin(\omega t + \frac{\pi}{2}) = y_0\cos(\omega t)$. 12. $y(t) = \frac{v_0}{\omega}\sin(\omega t) + y_0\cos(\omega t)$.

Answers to Selected Homework 18 Problems. 2. (a) $y_0 = D - Ak\alpha/(k^2+\alpha^2)$, (b) for k small the time delay is nearly 6 hours and the amplitude is small, while for k large the time delay is small and the amplitude is only a little less than A, (c) for k near zero, time delay$= \frac{1}{\alpha}\Big(\pi/2 - \arctan\big(\frac{k}{\alpha}\big)\Big) \approx 6 - k/\alpha^2 = 6 - 144k/\pi^2$ (since $\arctan(x) \approx x$ if x is near 0), while for k large, time delay$= \frac{12}{\pi}\arctan\big(\frac{\pi}{12k}\big) \approx 1/k$, (d) $\theta = 60°$, so time delay is 4 hrs, hottest: 7 p.m.
3. $dy/dt = -0.01y + (2 + 0.25t)$. 4. $\frac{dy}{dt} = -k\big(y - (b+at)\big)$.
5. $dy/dt = -2y/(100 + 0.25t^2) + (4+t)$. 6. $dy/dt = -\frac{r}{V}y + r(k+\alpha t)$.
7. (a) $dy/dt = ky + \big(18.5\sin\big(\frac{2\pi t}{365}\big) - 20\big)$, (b) \$29200, (c) you can use the solution to Example 18.2 by replacing k by $-k$ throughout Example 18.2, where $k = 0.00025\,\text{day}^{-1}$, and choosing $\alpha = -2\pi/365$, $\theta = \arctan(2\pi/365k) = 1.5563$ rad, $D = 80000$, $A = 7400$; then $y(t) = 80000 + (y_0 - 78925.5)e^{0.00025t} - 1074.58\sin\big(\frac{2\pi}{365}t + 1.5563\big)$; (d) \$24136.
8. (a) $r(t) = 12 - at$, $V(t) = 100 - 12t + \frac{1}{2}at^2$; (b) $\frac{dy}{dt} = -\frac{r(t)}{V(t)}y + l$.
9. (a) $x_0e^{-k_1t}$, (b) $dy/dt = -k_2y + k_1x_0e^{-k_1t}$.
10. (a) $x(t) = kV + (x_0 - kV)e^{-rt/V}$, (b) $dy/dt = -\frac{r}{W}y + \big(rk + \big(\frac{x_0}{V} - k\big)re^{-rt/V}\big)$.

1. Answers to Practice Midterms

Answers to Practice First Midterm #1

1. $v(t) = -1.5 + \int_0^t \sin(u/3)du = 1.5 - 3\cos(t/3)$; velocity is at first negative, then becomes positive after $v(t)$ crosses the t-axis, i.e., when: $1.5 - 3\cos(t/3) = 0$; $\cos(t/3) = 0.5$; $t/3 = \pi/3$; $t = \pi$. So total distance $= -\int_0^\pi (1.5 - 3\cos(t/3))dt + \int_\pi^5 (1.5 - 3\cos(t/3))dt = = -(1.5t - 9\sin(t/3))|_0^\pi + (1.5t - 9\sin(t/3))|_\pi^5 = -1.5\pi + 9(\sqrt{3}/2) + 7.5 - 9\sin(5/3) - (1.5\pi - 9(\sqrt{3}/2)) = -3\pi + 9\sqrt{3} + 7.5 - 9\cdot 0.9954 = 4.705$.
2.

$$\text{(a) } \pi\int_0^{1/2}\cos^2(\pi x) - (1-2x)^2dx; \qquad \text{(b) } \overline{x} = 0,\ \overline{y} = \frac{\int_0^4 1.5x^2dx + \int_4^5 2x\sqrt{25-x^2}dx}{\int_0^4 1.5x\,dx + \int_4^5 2\sqrt{25-x^2}dx}.$$

3. (a) $\frac{1}{5}\cdot\pi\int_{-3}^0 x^2dy = \frac{\pi}{5}\int_{-3}^0 25(1 - y^2/9)dy = 5\pi(y - y^3/27)|_{-3}^0 = 10\pi$ sec.
(b) 5/(cross-sectional area)$= 5/(\pi 5^2) = 0.2/\pi$ units/sec; the average rate is total rise divided by T, i.e., $3/10\pi = 0.3/\pi$, which is 1 1/2 times as great.

4. (a) $r\int_0^\pi \sqrt{(1 - 0.5\cos\theta)^2 + (0.5\sin\theta)^2}\,d\theta = r\int_0^\pi \sqrt{1.25 - \cos\theta}\,d\theta$.
(b) $r\frac{\pi}{6}\Big(\frac{1}{2}\cdot 0.5 + \sqrt{1.25 - 0.5\sqrt{3}} + \sqrt{1.25 - 0.5} + \sqrt{1.25} + \sqrt{1.25 + 0.5} + \sqrt{1.25 + 0.5\sqrt{3}}$
$+\frac{1}{2}\cdot 1.5\Big) = r\frac{\pi}{6}(0.25 + 0.6197 + 0.8660 + 1.1180 + 1.3229 + 1.4547 + 0.75) = 3.3412r$.

Answers to Practice First Midterm #2

1. $x(t) = (10\cos\alpha)t$, $y(t) = -4.9t^2 - (10\sin\alpha)t + s_0$. Since $y(5) = 0$ and $x(5) = 25$, from the second equation we get $\cos\alpha = 0.5$, i.e., $\alpha = 60°$, and from the first equation we then get $0 = -4.9\cdot 5^2 - (10\cdot 0.5\sqrt{3})5 + s_0$, so that $s_0 = 165.8$ m.
2.

$$\text{(a) } \pi\int_0^2 \frac{1}{3-y}dy \qquad\qquad \text{(b) } \int_0^{0.1} 5\sqrt{(1-\cos(12.5t))^2 + \sin^2(12.5t)}\,dt.$$

3. (a) Taking d/dt of $\frac{x^2}{25}+\frac{y^2}{16}=1$ gives $y\,\dot{y}=-0.64x\,\dot{x}=-0.384$. When $x=3$, we have $y=3.2$, so that $\dot{y}=-0.384/3.2=-0.12$. (b) The position of the shadow is $x+y$, so that its rate is $\dot{x}+\dot{y}=0.20-0.12=0.08$. (c) $\sqrt{\dot{x}^2+\dot{y}^2}=\sqrt{0.20^2+0.12^2}=0.2332$.

4. (a) $\int_0^\pi \sin(x)dx \Big/ \int_0^\pi \frac{\sin x}{x}dx$. Top integral $=2$.

(b) Bottom integral $=\frac{\pi}{6}\left(\frac{1}{2}\cdot 1+\frac{0.5}{\pi/6}+\frac{0.5\sqrt{3}}{\pi/3}+\frac{1}{\pi/2}+\frac{0.5\sqrt{3}}{2\pi/3}+\frac{0.5}{5\pi/6}+0\right)=1.84465$.

(c) $\overline{x}=1.0842$.

Answers to Practice First Midterm #3

1. $y=2\sin\left(\frac{\pi}{6}(x-1)\right)$, $\int_2^6\left(2\sin\left(\frac{\pi}{6}(x-1)\right)-1\right)dx=12\sqrt{3}/\pi\ -\ 4=2.616$.

2. (a) $\int_0^1\sqrt{13.0^2+(-9.8t-7.5)^2}dt$.

(b) $c=ea=6$, so $b=\sqrt{a^2-c^2}=8$, $4\int_0^{\pi/2}\sqrt{100\sin^2\theta+64\cos^2\theta}\,d\theta$.

(c) $\int_0^2\left(10-\frac{10}{0.1t+1}\right)dt$.

3. $\int_0^3\sqrt{\dot{x}(t)^2+\dot{y}^2}dt\approx 65.98$ meters (in practical terms 66 meters, since your data are given to only two places of accuracy).

4. (a) $x=3t$, $y=2-2t$. (b) $\sin\theta$ in x-direction, $\cos\theta$ in y-direction.

(c) $\theta=\pi t/2$. (d) $x=3t+\sin(\pi t/2)$, $y=2-2t+\cos(\pi t/2)$.

Answers to Practice First Midterm #4

1. $\int_0^1\cos(\pi x/2)dx-\int_0^1(1-x)dx=(2/\pi)-(1/2)=0.13662$.

2. (a) $\int_0^{0.7}\sqrt{6.86^2+(-9.8t+6.86)^2}dt$. (b) $\pi\int_0^2 9(1-y^2/4)dy$.

3. $\overline{x}=\int_0^{\pi/2}x\sin x\,dx\ \Big/\ \int_0^{\pi/2}\sin x\,dx$, bottom$=1$,

top$\approx\frac{\pi}{12}\left(\frac{\pi}{12}\sin\frac{\pi}{12}+\frac{\pi}{6}\sin\frac{\pi}{6}+\frac{\pi}{4}\sin\frac{\pi}{4}+\frac{\pi}{3}\sin\frac{\pi}{3}+\frac{5\pi}{12}\sin\frac{5\pi}{12}+\frac{1}{2}\cdot\frac{\pi}{2}\right)=1.00573$, so $\overline{x}=1.00573$.
(Note: Later, when we come to integration by parts, we will be able to compute the top integral exactly, and find that it is actually equal to 1.)

4. (a) linear distance car traveled $=3t^2+20t$, angular distance $\theta=15t^2+100t$ radians. (b) About 20° beyond the top point (clockwise). (c) $dx/dt=(6t+20)(1-\cos(15t^2+100t))=44.661$ when $t=0.5$, $dy/dt=(6t+20)\sin(15t^2+100t)=-7.734$ when $t=0.5$, so speed$=\sqrt{44.661^2+7.734^2}=45.33$ m/sec.

Answers to Practice First Midterm #5

1. Note that $\overline{y}=\overline{x}$ because of symmetry; the bottom integral in $\overline{x}$ is $\int_0^1(0.5x+1)dx+\int_1^2(0.5x+1-(2x-2))dx=5/4+3/4=2$; the top integral in $\overline{x}$ is $\int_0^1 x(0.5x+1)dx+\int_1^2 x(0.5x+1-(2x-2))dx=\int_0^1(0.5x^2+x)dx+\int_1^2(-1.5x^2+3x)dx=5/3$, so that $\overline{x}=(5/3)/2=5/6$.

2. (a) $c=\sqrt{a^2-b^2}=24$, foci at $(-24,0)$ and $(24,0)$. (b) $e=c/a=24/25=0.96$. (c) $x^2/25^2+y^2/7^2=1$, so $x^2=625(1-y^2/49)$, which is $625\cdot 33/49$ when $y=-4$; $\frac{dh}{dt}=\frac{dV/dt}{dV/dh}=\frac{0.7}{\pi\cdot 625\cdot 33/49}=0.00052936$ m/sec.

3. (a) $x=6t-0.3\sin(15t)$, $y=0.4-0.3\cos(15t)$. (b) When $t=0.4$, $x=2.4838$ and $y=0.1119$, which is below and slightly to the right of the center, whose coordinates at that time are $(2.4,0.4)$. (c) $dx/dt=1.6792$, $dy/dt=-1.2573$, speed$=2.098$ m/sec directed $36.8°$ below the horizontal.

4. (a) $v_{\text{horiz}}(t)=\frac{1}{2}t^2$, $v_{\text{vert}}(t)=\frac{6}{\pi}\sin(\pi t/6)$. (b) $\int_0^3\sqrt{\left(0.5t^2\right)^2+\left(\frac{6}{\pi}\sin(\pi t/6)\right)^2}dt$.

(c) $\frac{1}{2}\left(\frac{1}{2}f(0)+f(0.5)+f(1)+f(1.5)+f(2)+f(2.5)+\frac{1}{2}f(3)\right)$, where
$f(t)=\sqrt{0.25t^4+(36/\pi^2)\sin^2(\pi t/6)}$ (final answer $=6.007$).

Answers to Practice Second Midterm #1

1. If you take 2 as the hypotenuse of the triangle and x as the leg opposite θ, then $x = 2\sin\theta$, $dx = 2\cos\theta\,d\theta$, $\sqrt{4-x^2} = 2\cos\theta$, and the limits of integration become 0 to $\pi/6$. The top integral in $\overline{x}$ becomes: $\int_0^{\pi/6} 8\sin^3\theta\,d\theta = 8\int_0^{\pi/6}\left(1-\cos^2\theta\right)\sin\theta\,d\theta = 8\int_1^{\sqrt{3}/2}(1-u^2)(-du)$ (where $u = \cos\theta$). This leads to the value $\frac{16}{3} - 3\sqrt{3} = 0.13718$ for the top integral. The bottom integral is: $\int_0^{\pi/6} 4\sin^2\theta\,d\theta = 4\int_0^{\pi/6}\left(\frac{1-\cos(2\theta)}{2}\right)d\theta = \left(2\theta - \sin(2\theta)\right)\Big|_0^{\pi/6} = \frac{\pi}{3} - \frac{\sqrt{3}}{2} = 0.18117$. So $\overline{x} = 0.13718/0.18117 = 0.75718$.

2. Since $x^4/(4-x^2) = -x^2 - 4 + 16/(4-x^2) = -x^2 - 4 - \frac{16}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)$, we have:
$\text{Vol} = \pi\int_0^1 \left(-x^2 - 4 - \frac{16}{4}\left(\frac{1}{x-2} - \frac{1}{x+2}\right)\right)dx = \pi\left(-\frac{x^3}{3} - 4x - 4\ln\left|\frac{x-2}{x+2}\right|\right)\Big|_0^1 = \pi\left(-\frac{13}{3} + 4\ln 3\right)$
$= 0.1920$.

3. Vol$= f(x) = \pi(e^{-x})^2 x = \pi x e^{-2x}$. Set $0 = f'(x) = \pi\left(-2xe^{-2x} + e^{-2x}\right) = \pi e^{-2x}(-2x+1)$, so $-2x+1 = 0$, i.e., $x = 1/2$. So the maximum volume is $\pi\cdot\frac{1}{2}\cdot e^{-1} = \pi/2e = 0.57786$.

4. (a) $(x_A, y_A) = (8t, 40-4.9t^2)$, (b) $10-2t$, (c) $\arctan\big((40-4.9t^2)/8t\big) - Arcsin\big((10-2t)/20\big)$, (d)

$$\frac{-8t(9.8t) - 8(40-4.9t^2)}{(8t)^2 + (40-4.9t^2)^2} + \frac{0.1}{\sqrt{1-(0.5-0.1t)^2}}.$$

Answers to Practice Second Midterm #2

1. (a) $f(1) = 0$ and $\lim_{x\to 0} f(x) = 0$. (b) $f'(x) = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}} = 0$ gives $\ln x = -2$, so $x = 1/e^2$ and the point is $(1/e^2,\ -2/e)$. (c) The top integral is

$$\int_0^1 x^{3/2}\ln x\,dx = \frac{2}{5}x^{5/2}\ln x\,\Big|_0^1 - \int_0^1 \frac{2}{5}x^{5/2}\cdot\frac{1}{x}\,dx = -\frac{2}{5}\int_0^1 x^{3/2}dx = -\frac{4}{25}.$$

Similarly, the bottom integral is $\int_0^1 x^{1/2}\ln x\,dx = -\frac{4}{9}$. So $\overline{x} = 9/25 = 0.36$.

2. (a) Write $1/(x^2+5x+4) = \frac{1}{-1-(-4)}\left(\frac{1}{x+1} - \frac{1}{x+4}\right)$, so we get $\frac{1}{3}\ln\left(\frac{x+1}{x+4}\right)\Big|_0^\infty = \frac{1}{3}\ln 4 = 0.4621$. (b) Write $x^2+2x = (x+1)^2 - 1^2$; if you take $x+1$ for the hypotenuse of the triangle and 1 for the leg adjacent to θ, then you have $x+1 = \sec\theta$, $dx = \sec\theta\tan\theta\,d\theta$, $\sqrt{x^2+2x} = \tan\theta$, and the limits of integration become 0 to $\pi/2$ (because the triangle has $\theta = 0$ when $x = 0$ and has angle θ approaching $\pi/2$ as $x \longrightarrow \infty$). In the integral everything cancels, and you get simply $\int_0^{\pi/2} d\theta = \pi/2 = 1.5708$.

3. $\pi\int_0^{\pi/2} x\cos^2 x\,dx = \pi\int_0^{\pi/2} x\frac{1+\cos(2x)}{2}dx = \pi\left(\frac{\pi^2}{16} + \frac{1}{2}\int_0^{\pi/2} x\cos(2x)dx\right) =$
$= \frac{\pi^3}{16} + \frac{\pi}{2}\left(\frac{1}{2}x\sin(2x)\,\Big|_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2}\sin(2x)dx\right) = \frac{\pi^3}{16} - \frac{\pi}{4} = 1.1525.$

4. $y_A(t) = 5e^{0.03t}$, $y_B(t) = 15e^{-0.05t}$, so minimize $f(t) = 5e^{0.03t} + 15e^{-0.05t}$. Set $0 = f'(t) = 0.15e^{0.03t} - 0.75e^{-0.05t}$, so $0.15e^{0.03t} = 0.75e^{-0.05t}$, and $e^{0.03t}/e^{-0.05t} = 0.75/0.15$, i.e., $e^{0.08t} = 5$. Take ln of both sides and divide by 0.08 to get $t = (\ln 5)/0.08 = 20.118$ hours.

Answers to Practice Second Midterm #3

1. (a) $\int_0^1 (1+2y)\arctan(y)dy = (y+y^2)\arctan(y)\,\Big|_0^1 - \int_0^1 \frac{y+y^2}{1+y^2}dy = \frac{\pi}{2} - \int_0^1\left(1 + \frac{y}{1+y^2} - \frac{1}{1+y^2}\right)dy$
$= \frac{\pi}{2} - \left(y + \frac{1}{2}\ln(1+y^2) - \arctan\ y\right)\Big|_0^1 = \frac{3}{4}\pi - 1 - \frac{1}{2}\ln 2.$
(b) Write $x - x^2 = 0.5^2 - (x-0.5)^2$, take 0.5 as the hypotenuse and $x-0.5$ as the leg opposite θ; then $\int\frac{x\,dx}{\sqrt{x-x^2}} = \int\frac{0.5(\sin\theta+1)0.5\cos\theta\,d\theta}{0.5\cos\theta} = -0.5\cos\theta + 0.5\theta + C = -\sqrt{x-x^2} + \frac{1}{2}Arcsin(2x-1) + C$.

(c) $\int_{\pi}^{4\pi} \cos^3(t/4)dt = \int_{\pi}^{4\pi} \left(1-\sin^2(t/4)\right)\cos(t/4)dt = \int_{\sqrt{2}/2}^{0}(1-u^2)(4\,du)$ (where $u = \sin(t/4)$); this is computed to be $-5\sqrt{2}/3$.

2. $\ln y = \ln\left((\ln x)^{e^x}\right) = e^x \ln(\ln x)$, which leads to $\frac{1}{y}\frac{dy}{dx} = e^x\left(\frac{1}{x\ln x} + \ln(\ln x)\right)$, and then $\frac{dy}{dx} = (\ln x)^{e^x}\left(\frac{1}{x\ln x} + \ln(\ln x)\right)$.

3. (a) $dv/dt = -kv$; (b) $v(t) = 5e^{-kt}$; (c) $s(t) = \int_0^t 5e^{-ku}du = \frac{5}{k}\left(1-e^{-kt}\right)$; (d) (1) $5e^{-0.8} = 2.247$ m/sec, (2) $50(1-e^{-0.8}) = 27.53$ m, (3) $s/r = 137.67$ rad, (4) $v/r = 11.23$ rad/sec; (e)

$$\frac{dL}{dt} = \frac{dL}{d\theta}\,\frac{d\theta}{dt} = \left(\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\right)\cdot 11.23 = 11.23\sqrt{r^2(1-\cos\theta)^2 + r^2\sin^2\theta} = 1.247\,\frac{\text{m}}{\text{sec}}$$

when $r = 0.2$, $\theta = 137.67$ rad.

Answers to Practice Second Midterm #4

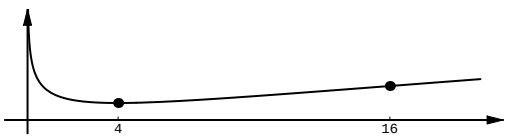
1. (a) $\int_{10\sqrt{3}}^{\infty} \frac{1}{100}\frac{dx}{1+(x/10)^2} = \frac{1}{10}\int_{\sqrt{3}}^{\infty}\frac{du}{1+u^2} = \frac{1}{10}\arctan u\,\Big|_{\sqrt{3}}^{\infty} = \frac{1}{10}\left(\frac{\pi}{2}-\frac{\pi}{3}\right) = \pi/60$.

(b) $\int (x+2)\left(\frac{1}{x-2}-\frac{1}{x-1}\right)dx = \int\frac{x+2}{x-2}dx - \int\frac{x+2}{x-1}dx = \int\left(1+\frac{4}{x-2}\right)dx - \int\left(1+\frac{3}{x-1}\right)dx = 4\ln|x-2| - 3\ln|x-1| + C$. (c) Set $u = 2(x+1)$, so that:
$\int_{-1/2}^{1/2} x\,e^{2(x+1)}dx = \frac{1}{2}\int_1^3(\frac{1}{2}u-1)e^u du = \frac{1}{4}\int_1^3 ue^u du - \frac{1}{2}\int_1^3 e^u du = \frac{1}{4}\left(ue^u|_1^3 - e^u|_1^3\right) - \frac{1}{2}e^u|_1^3 = \frac{1}{2}e$.

(d) $\int_0^{\pi}\sin^5(x/3)dx = \int_0^{\pi}\left(1-\cos^2(\frac{x}{3})\right)^2\sin(\frac{x}{3})dx = \int_1^{1/2}(1-u^2)^2(-3\,du)$ (where $u = \cos(\frac{x}{3})$). This gives $3(-\frac{1}{5}u^5 + \frac{2}{3}u^3 - u)|_1^{1/2} = 0.331$.

2. (a) $\arctan\Big((7-t-2\sin(2\pi t))\;/\;(3+1.4t-2\cos(2\pi t))\Big)$. (b) Let $y(t)$ be the top function and $x(t)$ the bottom function in the Arctan. Then $\frac{d}{dt}\arctan\left(\frac{y}{x}\right) = \frac{1}{1+(y/x)^2}\cdot\frac{x\dot{y}-y\dot{x}}{x^2} = \frac{x\dot{y}-y\dot{x}}{x^2+y^2}$; multiply by $180/\pi$ to convert to degrees; the result is:

$$\frac{180}{\pi}\,\frac{(3+1.4t-2\cos(2\pi t))(-1-4\pi\cos(2\pi t)) - (7-t-2\sin(2\pi t))(1.4+4\pi\sin(2\pi t))}{(3+1.4t-2\cos(2\pi t))^2 + (7-t-2\sin(2\pi t))^2}.$$

3. (a) $0 = f'(x) = (1/2\sqrt{x}) - (1/x)$ gives $x = 4$, i.e., the point $(4, 0.6137)$. (b) $f''(x) = -\frac{1}{4}x^{-3/2} + x^{-2}$, which is positive if $x = 4$, so that the point in part (a) is a minimum. (c) $f''(x) = 0$ when $x = 16$, and the point $(16, 1.2274)$ is an inflection; the curve is concave up for $x < 16$ and concave down for $x > 16$. (d) $f(x)$ approaches $+\infty$ as $x \longrightarrow 0$ and as $x \longrightarrow \infty$.

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