Practice First Midterms

Practice First Midterm #1. (50 points in all, time = 1 hour)

1. (13 points) A particle is constrained to move only in the x-direction. A rotating force of constant magnitude causes it to accelerate in the x-direction with $a(t) = \sin(t/3)$. The particle's initial velocity is 1.5 units/sec **to the left**. Find the **TOTAL** distance the particle travels between $t = 0$ and $t = 5$ sec.

2. (12 points) Write each of the following in terms of definite integrals. Do **NOT** attempt to evaluate the integrals.

(a) The volume of the solid of revolution obtained by rotating around the x-axis the shape in the first quadrant bounded by the line $2x + y = 1$ and the curve $y = \cos(\pi x)$.

(b) The center of mass of a uniform plate in the 1st and 4th quadrants that is bounded by the line from the origin to $(4, 3)$, the line from the origin to $(4, -3)$, and the circumference of the circle of radius 5 centered at the origin.

3. (12 points) An ellipse is widest in the x-direction, where it measures 10 units across, and narrowest in the y-direction, where it measures 6 units across. The bottom half of this ellipse is rotated around its minor axis to form a container, into which water is poured at the rate of 5 cubic units/sec.

(a) Find the length of time T before the container is filled.

(b) At time T (i.e., the instant when the container is about to overflow) find the rate at which the water level is rising. Compare this with the *average* rate at which the water level rose between time $t = 0$ and $t = T$.

4. (13 points) A pebble is caught half-way up the spoke of a wheel of radius r. The wheel is rolling along a flat surface without slipping.

(a) Set up a definite integral for the distance along its path that the pebble travels as the wheel goes through half a revolution, during which time the pebble goes from a position directly below the center of the wheel to a position directly above the center.

(b) Find this distance using the trapezoid rule with $n = 6$ subdivisions. Express your answer as a multiple of r. To receive credit you must show clearly the steps in applying the trapezoid rule, indicating what numbers you are substituting into the rule.

Practice First Midterm #2. (50 points in all, time = 1 hour)

1. (12 points) In this problem neglect air resistance, choose upward to be the positive direction, and take $g = 9.8$ m/sec^2. You know that a ball was thrown down from a window at 10 m/sec, but you do not know the angle α below the horizontal at which it was thrown. You also do not know the height s_0 of the window. Suppose that the ball hits the ground after 5 seconds at a point on the ground that is 25 meters from the side of the building from which the ball was thrown. Find α and s_0.

2. (12 points) Write each of the following in terms of definite integrals. Do **NOT** attempt to evaluate the integrals.

(a) The volume of the solid of revolution obtained by rotating around the y-axis the shape bounded by the x-axis, the line $y = 2$, and the curve $y = 3 - (1/x^2)$.

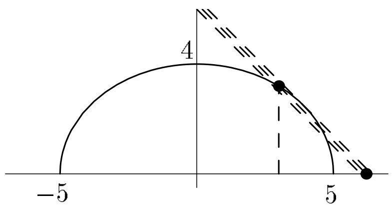
(b) The distance along its path traveled in the first tenth of a second by a pebble which at time $t = 0$ is caught in the tread of a wheel of radius 40 cm that rolls along the pavement without slipping at 5 m/sec.

3. (14 points) A bug is on the circumference of the ellipse pictured at the right. It crawls in such a way that its x-coordinate increases at the rate of 0.20 units/sec. The sun is shining from the left from an angle of 45° above the horizontal, casting the bug's shadow on the x-axis. At the instant when the x-coordinate of the bug is 3, find:

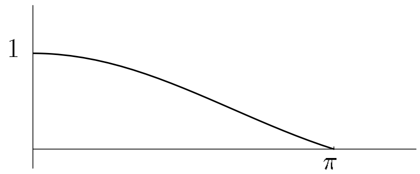
(a) the rate at which its y-coordinate is changing,

(b) the rate at which its shadow is moving along the x-axis,

(c) the speed of the bug along the ellipse.



4. (12 points) The graph to the right shows the function $f(x) = (\sin x)/x$ between $x = 0$ and $x = \pi$. Actually, when $x = 0$ the ratio $(\sin x)/x$ is $0/0$, which is undefined; however, the expression $(\sin x)/x$ has the limit 1 as x approaches 0, so we take $f(0) = 1$. A uniform plate is bounded by $f(x)$ and the x- and y-axes.



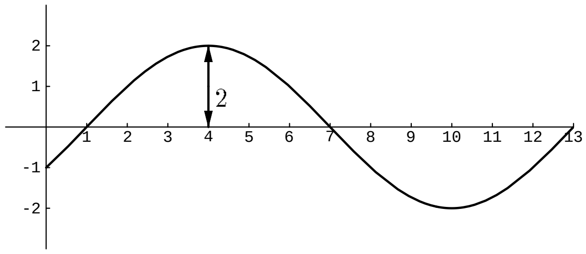
(a) Write the x-coordinate $\overline{x}$ of its center of mass in terms of definite integrals. One of the two definite integrals is easy to evaluate by the anti-derivative method.

(b) Use the trapezoid rule with $n = 6$ subdivisions to evaluate the second of the two definite integrals in part (a). To receive credit you must show clearly the steps in applying the trapezoid rule, indicating what numbers you are substituting in the rule.

(c) Find $\overline{x}$.

Practice First Midterm #3. (50 points in all, time = 1 hour)

1. (10 points) Give the equation of the sinusoidal function below, and find the area of the region between the portion of the curve shown and the line $y = 1$.



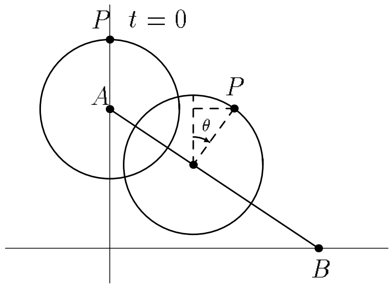
2. (20 points) Set up each of the following in terms of definite integrals. Do **NOT** evaluate the integrals.

(a) The distance along its trajectory traveled in the first second by an object thrown at 15 m/sec directed 30° *below* the horizontal from a window 20 meters high. (Neglect air resistance, take upward to be the positive direction, and take $g = 9.8$ m/sec^2.)

(b) The entire circumference of an ellipse with semimajor axis $a = 10$ and eccentricity $e = 0.6$. (First find b, and write the ellipse in parametric form.)

(c) The net distance traveled between time $t = 0$ and $t = 2$ if the acceleration is $a(t) = 1/(0.1t + 1)^2$ and the initial velocity is 0.

3. (20 points) A wheel of radius 1 rotates clockwise at the rate of 1 revolution every four seconds as the center of the wheel moves along the straight line joining the points $A = (0, 2)$ and $B = (3, 0)$ (see the drawing at the right). The center moves at constant velocity, starting at A at time $t = 0$ and ending up at B at time $t = 1$. P is the point on the rim of the wheel which is directly above the center at time 0.



(a) Find parametric formulas in terms of t for the motion of the center of the wheel.

(b) After the wheel has rotated through an angle of θ, how far is the point P from the center in the x-direction and in the y-direction?

(c) Express θ in terms of t.

(d) Find parametric formulas in terms of t for the motion of P.

(e) Write an integral for the distance traveled by the point P between time 0 and 1.

(f) Using the trapezoid rule with $n = 4$ subdivisions, evaluate the integral in part (e). Please show your work carefully.

Practice First Midterm #4. (50 points in all, time = 1 hour)

1. (8 points) Find the area in the first quadrant between the graphs of $y = \cos(\pi x/2)$ and $x + y = 1$.

2. (16 points) Set up each of the following in terms of definite integrals. Do **NOT** evaluate the integrals.

(a) The distance along its trajectory until reaching the peak of the trajectory that is traveled by an object thrown at 9.7 m/sec directed 45° above the horizontal. (Neglect air resistance, take upward to be the positive direction, and take $g = 9.8$ m/sec^2.)

(b) The volume of the solid obtained by rotating around the y-axis the region in the first quadrant bounded by the axes and the curve which is given parametrically by the formulas $x = 3\cos\theta$, $y = 2\sin\theta$.

3. (12 points) Find the center of mass of a uniform flat plate bounded by $y = \sin x$, $y = -\sin x$, and $x = \pi/2$. Use the trapezoid rule with $n = 6$ subdivisions to find the value of the top integral. Please show your work clearly.

4. (14 points) In this problem you will need the parametric formulas for a cycloid: $x = r(\theta - \sin(\theta))$, $y = r(1 - \cos(\theta))$. A car accelerates at the constant rate of 6 m/sec^2. When it reaches a speed of 20 m/sec, it picks up a pebble in the tread of a wheel. The wheel has radius 20 cm. Let t denote the time measured from the instant the wheel picks up the pebble.

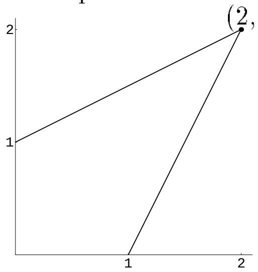
(a) Express in terms of t the angle θ through which the pebble has rotated around the center of the wheel. (θ is measured in radians.)

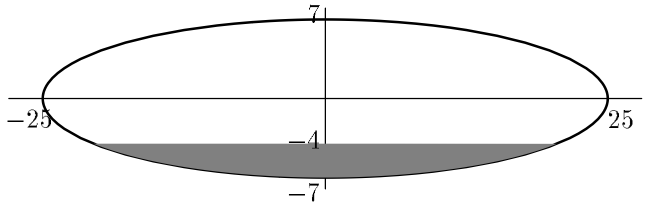
(b) Draw a circle with an arrow pointing to the position of the pebble on the wheel when $t = 0.5$ sec.

(c) Find the speed of the pebble along its trajectory when $t = 0.5$ sec.

Practice First Midterm #5. (50 points in all, time = 1 hour)

1. (12 points) Find the coordinates of the center of mass of a uniform flat plate in the shape shown below to the left.





2. (12 points) The ellipse pictured above to the right is centered at the origin of the xy-plane. Its semi-major axis is 25 and its semi-minor axis is 7. The units are meters.

(a) Find the coordinates of its foci.

(b) Find the eccentricity.

(c) Suppose that the bottom half of the ellipse is rotated around the y-axis to form a container. Water is entering the container at the rate of 0.7 m^3/sec. How fast is the water level rising when the water is 3 meters deep at its deepest point (i.e., when the water level is 4 meters below the top of the container)?

3. (12 points) A bicycle has a wheel of radius 0.4 meter. It is traveling at a constant speed of 6 m/sec. A pebble is lodged in a spoke of the wheel at a distance of 0.3 m from the center. At time $t = 0$ the pebble is at its lowest point (0.1 m above the pavement). Suppose that the wheel is moving from left to right along the x-axis, and the origin is the point where the wheel meets the pavement at time $t = 0$. In this problem, use the parametric formulas $x = r\theta - a\sin\theta$, $y = r - a\cos\theta$ for the coordinates of the pebble after the wheel has rotated through an angle of θ radians.

(a) Find formulas for the coordinates of the pebble at time t.

(b) When $t = 0.4$ sec, find the coordinates of the pebble, and draw a diagram showing the relative positions of the center of the wheel and the pebble at that instant.

(c) When $t = 0.4$ sec, find the speed of the pebble along its trajectory, and draw a picture of the velocity vector showing the pebble's direction of motion.

4. (14 points) At time $t = 0$ a particle is at rest (zero velocity) at the origin of the xy-plane. In the horizontal direction its acceleration at time t is given by the formula $a_{\text{horiz}}(t) = t$, and in the vertical direction its acceleration at time t is given by the formula $a_{\text{vert}}(t) = \cos(\pi t/6)$.

(a) Find formulas for the horizontal velocity and the vertical velocity at time t.

(b) Express the arclength of the particle's path during the first 3 seconds (the distance it has traveled) as a definite integral.

(c) Use the trapezoid rule with $n = 6$ to find the value of the integral in part (b). You do **NOT** have to do the final numerical computations, but be sure you show clearly the values that you are substituting into the trapezoid rule formula.