Practice Second Midterms

Practice Second Midterm #1. (50 points in all, time = 1 hour)

1. (15 points) A shape is bounded by the curve $y = x^2/\sqrt{4 - x^2}$, the x-axis, and the line $x = 1$. Find the x-coordinate of the center of mass of a uniform plate in that shape. Please show all your work carefully (including the triangle you use for the inverse trig substitution, and the trig identities you use to integrate powers of sine or cosine).

2. (10 points) The shape in Problem 1 is rotated around the x-axis. Find the volume of the resulting solid of revolution. Again please show all of your work.

3. (10 points) Find the maximum possible volume of the cylinder obtained by rotating the rectangle shown at the right around the x-axis. (The upper-right corner of the rectangle is at the point (x, e^{-x}) on the curve $y = e^{-x}$. Volume of cylinder $= \pi r^2 h$.)

4. (15 points) At time $t = 0$ a ball is thrown horizontally at 8 m/sec from a point 40 meters directly above the observer's eye O. (See the drawing at the right, neglect air resistance, and take $g = 9.8$ m/sec^2.) At the same instant a 20-meter span of a bridge extends from O at an angle of 30° above the horizontal. The bridge is being lowered by a rope (the vertical dotted line) which is pulled downward at the steady rate of 2 m/sec.

(a) Express the x- and y-coordinates of the ball as functions of time.

(b) Express the height of the point B (the tip of the bridge span) in terms of t.

(c) Express the angle AOB (the angle between the observer's line of vision to the ball and her line of vision along the bridge) in terms of t.

(d) Find a formula for the rate in rad/sec at which the angle AOB is decreasing at time t.