Lab Project II

Two identical jars of boiling water, labeled A and B, stand beside two identical jars of ice water , labeled C and D, in a room whose temperature remains constant. The contents of jars A and C are immediately mixed. Jars B and D are kept separate for a few minutes and then mixed. How will the temperatures of the two mixtures compare? One school of thought says that the AC mixture will be hotter because jar D, being very hot will lose its heat much more rapidly than the cooler AC mixture and the result will be that more heat is lost by B alone than by the AC misture. Another says that the hot water from A and the cold water from C will act independently, just like the separate portions in B and D and therefore the mistures will be the same temperature.

Run a number of (theoretical) experiments, varying the temperatures of the room and water, the quantities of hot and cold water, and the times. Prepare a report on the experiments, complete with an explanation of the calculations (the theory) , a separate section of results and a third of interpretation of the results relative to the original question. In a final section provide a theoretical explanation of the results of the experiments.

Alternative Formulation

Ms. Karen Heavin suggests the following alternative statement for the lab. You may report on the original, this, or your own version.

There were once three friends, Carl, Paul, and John. In an attempt to survive the last two weeks of the semester they had begun a nightly ritual of studying into the wee hours of the morning, replacing sleep with coffee. During one of their sessions, after consuming about 15 cups of coffee each, Carl decided to go run around the block to control the shaking of his limbs from too much coffee. Just as he was about to get up, John inadvertently poured Carl a full cup of fresh coffee. Carl said not to worry about it he would drink it when he got back. John then offered to put cream in the coffee so it would be "just right" when Carl got back from his run. Carl objected strenuously to this stating that the creme would cause his coffee to be too cold when he got back. Upon hearing this Paul assured Carl his coffee would not be any colder than if he left it alone and poured the cream in when he got back. John felt Paul was in error and told him so.

Anyway, Carl started to prepare for his run but fell asleep as he sat on his bed to put on his running shoes. John and Paul argued all night over the timing the cream pouring (rather than studying calculus).

The next morning Carl found his friends still arguing. However, after a nights sleep he could think straight. He drank the cold cup of coffee, then quickly put the argument to rest by modeling the problem with Newton's law of cooling (or whatever), doing some exprimental calculations, looking at some graphs and then explaining the results. Then Carl went off to school while John and Paul fell asleep.

GETTING STARTED: One will need to assume that the constant, k , in Newton's law of cooling is the same for all aspects of the problem. In addition one will need to assume that the temperature of a mixture of two containers of water will be the weighted average of the temperatures of the two components.

The following takes one through a related problem and contains all of the Maple commands and ideas one might need.

Problem : Two cups of soup, the first at 90 C and the second at 100 C are put in a room where the temp is maintained at 20 C. The first cup cooled from 90 C to 60 C after 10 minutes, at which time the second cup is put in a freezer at -5 C. Assume Newton's law of cooling and answer the following questions.

(a) Express the temperature of each cup as a function of time, and draw their graphs.

(b) How much longer will it take the two cups to reach the same temperature?

Solution: Assuming Newton's Law of cooling, the rate at which each cup cools is proportional to the difference between the ambient temperature and the temperature of the soup in the cup. The differential equation for each cup looks like this:

```
> restart;
> Diff(f(t),t) = k*(f(t)-20);
```

$$\frac{\partial}{\partial t}\mathrm{f}(t) = k\,(\mathrm{f}(t) - 20)$$

This is a separable differential equation, whose solution can be written in the form

```
> T := 20 + (T0-20)*exp(k*t);
```

$$T := 20 + (\mathit{T0} - 20)\,e^{(k\,t)}$$

where T = T0 at t=0 and k is determined by the information given that the first cup cools to 60 C in 10 minutes (i.e. T=60 at t=10).

So if we denote the temperature of the first cup after t minutes by f then

```
> f := subs(T0=90,T);
```

$$f := 20 + 70\,e^{(k\,t)}$$

```
> k :=solve(subs(t=10,f) =60,k);
```

$$k := \frac{1}{10}\ln(\frac{4}{7})$$

```
> f := unapply(f,t);
```

$$f := t \to 20 + 70\,e^{(1/10 \ln(4/7)\,t)}$$

Assume that this value of k is also valid for the second cup. (Why shouldn't it be? They are both soup.) So the second cup's temperature is given by

```
> g := subs(T0=100,T);
```

$$g := 20 + 80\,e^{(1/10 \ln(4/7)\,t)}$$

```
> g := unapply(g,t);
```

$$g := t \to 20 + 80\,e^{(1/10 \ln(4/7)\,t)}$$

for the first 10 minutes. From that time on, the second cup's temperature is given by

```
> h := t -> -5 + (itemp - (-5))*exp(k*t);
```

$$h := t \to -5 + (\mathit{itemp} + 5)\,e^{(k\,t)}$$

where itemp is the temperature that the 2nd cup would start at in order for it to cool to g(10) degrees C if it were in the freezer from the start. So itemp satisfies the equation

```
> eq := -5 + (itemp +5)*exp(k*10) = g(10);
```

$$eq := -\frac{15}{7} + \frac{4}{7}\,itemp = \frac{460}{7}$$

Solving for itemp,

```
> itemp := solve(eq,itemp);
```

$$itemp := \frac{475}{4}$$

Now plot the two temperature curves.

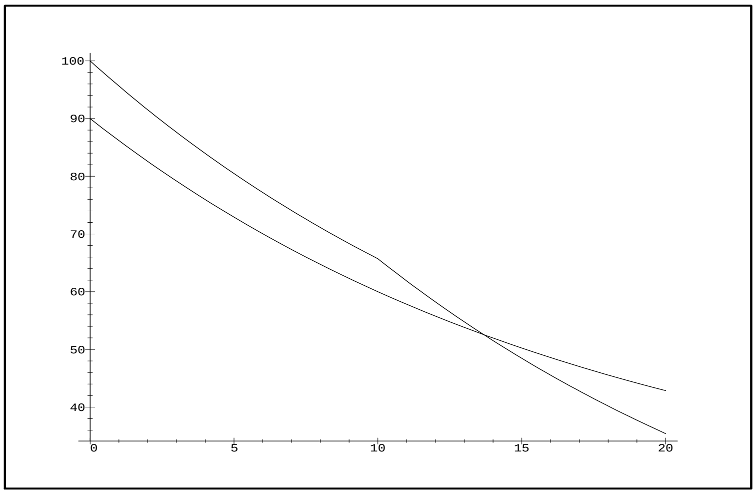
```
> cup1 := [t,f(t),t=0..20];
```

$$cup1 := [t,\, 20 + 70\, e^{(1/10 \ln(4/7)\, t)},\, t = 0..20]$$

```
> cup2 := [t,g(t),t=0..10],[t,h(t),t=10..20];
```

$$cup2 := [t,\, 20 + 80\, e^{(1/10 \ln(4/7)\, t)},\, t = 0..10],\ [t,\, -5 + \frac{495}{4}\, e^{(1/10 \ln(4/7)\, t)},\, t = 10..20]$$

```
> plot( {cup1,cup2},color=black );
```



The cups are at the same temperature before another 5 minutes pass.

```
> etime := solve(f(t)=h(t),t);
```

$$etime := 10\, \frac{\ln(\frac{20}{43})}{\ln(\frac{4}{7})}$$

```
> `Cups are same temp at t=`,evalf(etime),` minutes`;
```

$$\textit{Cups are same temp at } t =, 13.67845330, \quad \textit{minutes}$$

At that time the common temperature is

```
> f(etime);
```

$$\frac{2260}{43}$$

```
> evalf(f(etime)),` degrees celsius.`;
```

$$52.55813953, \quad \textit{degrees celsius.}$$