MA114 Worksheet for Section 11.1

Skills

1. Find a symbolic expresion for the general term a_k of each of the following sequences.

   (a) 1,3,5,7 ...

   (b) 1,4,7,10 ...

   (c) $\frac{2}{3}, \frac{4}{5}, \frac{8}{7}, \frac{16}{9}, \ldots$

   (d) $\frac{1}{2}, -\frac{1}{5}, \frac{1}{10}, -\frac{1}{17}, \frac{1}{26}, \ldots$

2. Calculate the first 5 terms for each of the following sequences, then determine if it converges or diverges. If it converges, find $\lim_{k\to\infty} a_k$.

   (a) $a_k = \left(\frac{1}{3}\right)^k$

   (b) $a_k = \left(\frac{-1}{2}\right)^k$

   (c) $a_k = (1.1)^k$

   (d) $a_k = \left(\frac{-3}{2}\right)^k$

   (e) $a_k = \arctan k$

   (f) $a_k = \frac{k^2}{k^2+k+3}$

   (g) $a_k = \sqrt{\frac{2k}{k+3}}$

   (h) $a_k = \frac{k}{\sqrt{k}+3}$

   (i) $a_k = \ln k - \ln(k+1) = \ln\left(\frac{k}{k+1}\right)$

   (j) $a_k = \frac{\cos k}{k+1}$

   (k) $a_k = e^{-k}\sin k$

   (l) $a_k = \int_0^k e^{-x}\,dx$

   (m) $a_k = 1 + \frac{(-1)^k}{k}$

   (n) $a_k = \frac{1+(-1)^k}{k}$

   (o) $a_k = 1 + (-1)^k$

   (p) $a_k = \frac{k+(-1)^k}{k}$

   (q) $a_k = \frac{k^2}{2k+1} - \frac{k^2}{2k-1}$

   (r) $a_k = \frac{k+\sin k}{k+\cos(4k)}$

   (s) $a_k = \frac{k^2}{e^k}$

   (t) $a_k = k\sin\frac{1}{k}$

(u) $a_k = \dfrac{k^2}{2k+1} \sin \dfrac{1}{k}$

(v) $a_k = \dfrac{\ln k}{k^3}$

(w) $a_k = (2^k + 3^k)^{1/k}$

3. Show that the sequence $.2, .22, .222, .2222, .22222, \ldots$ has a limit.

4. Let $a_k = \dfrac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots (2k)}$. Show that $\lim_{k\to\infty} a_k$ exists.

Concepts

1. Give an example of a sequence that is

   (a) convergent but not monotone.

   (b) bounded but not monotone.

   (c) monotone but not convergent.

   (d) unbounded but not monotone.

2. Let $a_n = \sum_{k=1}^{n} \dfrac{k}{n^2}$. Use Riemann sums to explain why $\lim_{n\to\infty} a_n = \int_0^1 x\,dx$.

3. Let $a_n = \sum_{k=1}^{n} \dfrac{k^2}{n^3}$. Evaluate $\lim_{n\to\infty} a_n$.

4. Let $a_n = \sum_{k=1}^{n} \dfrac{1}{n+k}$. Evaluate $\lim_{n\to\infty} a_n$.

5. Let a_1 and b_1 be two real numbers where $0 < b_1 < a_1$. Define sequences $\{a_n\}$ and $\{b_n\}$ as follows:

$$a_{n+1} = \frac{a_n + b_n}{2}$$
$$b_{n+1} = \sqrt{a_n b_n}$$

   (a) Show that $b_n < b_{n+1} < a_1$ for all n. Explain why this shows $\lim_{n\to\infty} a_n$ exists.

   (b) Show that $b_1 < a_{n+1} < a_n$ for all n. Explain why this shows $\lim_{n\to\infty} b_n$ exists.

   (c) Show that $0 < a_{n+1} - b_{n+1} < \dfrac{a_1 - b_1}{2^n}$. What does this show?

   (d) Evaluate $\lim_{n\to\infty} a_n$ and $\lim_{n\to\infty} b_n$.