MA114 Worksheet for Section 11.2

Skills

1. Determine whether each of the following series converges or diverges. If a series converges, find its limit. Justify your answers.

   (a) 2 - 2 + 2 -2 +2 -2 + $\cdots$

   (b) $\frac{3}{10} - \frac{3}{20} + \frac{3}{40} - \frac{3}{80} + \frac{3}{160} - \frac{3}{320} + \cdots$

   (c) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \frac{1}{5} - \cdots$

   (d) 1 -1 +2 -1 -1 +3 -1 -1 -1 +4 -1 -1 -1 -1 + $\cdots$

   (e) $\frac{4}{7^{10}} + \frac{4}{7^{12}} + \frac{4}{7^{14}} + \frac{4}{7^{16}} + \frac{4}{7^{18}} + \cdots$

   (f) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{2n+1}$

   (g) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{k+2} - \frac{1}{k+3} \right)$

   (h) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{3k} - \frac{1}{4k} \right)$

   (i) $\displaystyle\sum_{k=1}^{\infty} \left(\int_k^{k+1} \frac{1}{x^2}\, dx \right)$

   (j) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^n$

   (k) $\displaystyle\sum_{m=2}^{\infty} \frac{1}{(\ln 3)^m}$

   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\ln(3+n^2)}$

   (m) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4k^2-1}$ Hint: Use partial fractions.

2. Find the limit of each of the following convergent series.

   (a) 2 -5 + 9+ $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$

   (b) $\displaystyle\sum_{n=0}^{\infty} e^{-n}$

   (c) $\displaystyle\sum_{k=3}^{\infty} \left(\frac{e}{\pi} \right)^k$

   (d) $\displaystyle\sum_{j=5}^{\infty} \left(-\frac{1}{2} \right)^j$

   (e) $\displaystyle\sum_{j=0}^{\infty} \frac{3^j + 4^j}{5^j}$

3. For each of the following series, find all values of x for which the series converges.

   (a) $\displaystyle\sum_{k=0}^{\infty} x^k$

(b) $\displaystyle\sum_{m=2}^{\infty} \left(\frac{x}{5}\right)^m$

(c) $\displaystyle\sum_{n=3}^{\infty} (1+x)^n$

Concepts

1. Consider the series $\displaystyle\sum_{k=0}^{\infty} a_k = \sum_{k=0}^{\infty} \frac{1}{k+2^k}$.

   (a) Show that the sequence $\{S_n\}$ is increasing.

   (b) Explain why $a_k \leq 2^{-k}$ when $k \geq 0$.

   (c) Show that $S_n \leq 2 - 2^{-n} < 2$.

   (d) Show that the series $\displaystyle\sum_{k=0}^{\infty} a_k$ converges.

2. Consider the series $\displaystyle\sum_{k=0}^{\infty} a_k = \sum_{k=0}^{\infty} \frac{1}{2+3^k}$.

   (a) Show that the sequence $\{S_n\}$ is increasing and bounded above.

   (b) Does $\displaystyle\sum_{k=0}^{\infty} a_k$ converge? Justify your answer.

3. A ball is dropped from a height of 12 meters. Each time it strikes the ground, it bounces back to a height three-fourths the distance from which it fell. Find the total distance travelled by the ball before it comes to rest.

4. What is wrong with the following calculation?

   Let $S = 1 + 2 + 4 + 8 + \cdots$. Then

   $$2S = 2 + 4 + 8 + 16 + \cdots = -1 + (1 + 2 + 4 + 8 + \cdots) = S - 1.$$

   Therefore,

   $$S = -1.$$

Maple

1. The following series all converge. Use maple to approximate the limit. Choose larger and larger values of n until you are convinced S_n is accurate to 6 decimal places. After you have a numeric estimate, try using the *limit* command to see if maple can evaluate the series directly.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k!}$.

   (b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\sqrt{k^2-1}}$.

   (c) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{1}{k}$.