MA114 Worksheet for Section 11.3

Skills

1. Test each of the following series for convergence or divergence using the comparison test.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

   (b) $\displaystyle\sum_{k=2}^{\infty} \frac{\sqrt{k}}{k-1}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2^k)}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k-1)}}$

2. Test each of the following series for convergence or divergence using the integral test.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{1.01}}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k}$

   (c) $\displaystyle\sum_{k=1}^{\infty} ke^{-k}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{1+k^2}$

   (e) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)}$

   (f) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\sqrt{k^2-1}}$

   (g) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2+25}}$

3. Test each of the following series for convergence or divergence using the ratio test.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(2k-1)(2k+1)}{2^k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^k$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{10^k}{(2k)!}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{k!}$

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{3^k}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{e^k}{k^3}$

Concepts

1. Consider the series $S_n = \sum_{k=0}^{\infty} a_k$ where $a_k = \dfrac{1}{k+2^k}$.

   (a) Use the comparison test to show the series converges.

   (b) Compute an estimate of the limit of the series that is guaranteed to be within 0.001 of the exact value.

   (c) Is your estimate an overestimate or underestimate? Justify your anwer.

2. Show that each of the following series converges, then find an upper bound on the limit of each series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$

   (b) $\displaystyle\sum_{m=1}^{\infty} \frac{1}{m\sqrt{1+m^2}}$

3. For which values of p does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converge? Justify your answer.

4. Consider the series $\displaystyle\sum_{k=1}^{\infty} a_k = \frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2}\frac{1}{2^3} + \frac{1}{3^3} + \cdots$.

   (a) Explain why $\displaystyle\lim_{k\to\infty} \frac{a_{k+1}}{a_k}$ does not exist.

   (b) What does the ratio test say about the convergence of this series?

   (c) Show that the series converges and evaluate its limit.