MA114 Worksheet for Section 11.4

Skills

1. Determine whether each series is conditionally convergent, absolutely convergent, or divergent.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k-4}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k(k+1)}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{2^k}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\sqrt{k+3}}$

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sqrt{k}}{k^2+1}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\sqrt{k}}{k+1}$

   (g) $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1}\left(\frac{1}{5}\right)^k$

   (h) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^{k}}{k\ln k}$

   (i) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k!}{k^2 2^k}$

   (j) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\ln k}{k}$

   (k) $\displaystyle\sum_{k=1}^{\infty} \frac{(-2)^{k}}{3^k+k}$

   (l) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\arctan k}{k}$

2. Approximate the sum of each series accurate to within 0.001.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k}}{2k+1}\left(\frac{1}{3}\right)^k$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$

3. Find the least positive integer n such that S_n approximates the sum of the given convergent alternating series to within 0.0001; that is, find n such that $|S-S_n| < 10^{-4}$.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}\left(\frac{1}{3}\right)^k$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$

Concepts

1. Give an example of a convergent series $\displaystyle\sum_{n=1}^{\infty} a_n$ with the property that $\displaystyle\sum_{n=1}^{\infty} (a_n)^2$ diverges.

2. Suppose that $\displaystyle\sum_{m=1}^{\infty} a_m$ diverges. Is it possible that $\displaystyle\sum_{m=1}^{\infty} |a_m|$ converges? Justify your answer.

3. Show that the alternating harmonic series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ can be rearranged so that

   (a) the resulting series converges to 0.

   (b) the resulting series converges to 2.

   (See the blurb in Ostebee and Zorn titled **The wacky world of conditional convergence**).

4. Show that the series $e^{-x}\cos x + e^{-2x}\cos 2x + e^{-3x}\cos 3x + e^{-4x}\cos 4x + \cdots$ is absolutely convergent for all positive values of x. (Hint: Use the fact that $|\cos\theta| \le 1$).

5. What can you say about the convergence of the series below?

$$1 + r\cos\theta + r^2\cos 2\theta + r^3\cos 3\theta + \cdots$$

6. What is wrong with following argument?

$$A = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

$$\frac{1}{2}A = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \frac{1}{10} - \cdots$$

   Thus,

$$\frac{3}{2}A = A + \frac{1}{2}A = 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots$$

   The series on the right is a rearrangement of the terms of the series A, hence its sum must be A. So

$$\frac{3}{2}A = A,$$

   which implies that

$$A = 0.$$

   On the other hand, we can write

$$A = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \cdots$$

   This says that $A > 0$.