MA114 Worksheet for Section 11.5

Skills

1. Find the radius of convergence and interval of convergence for each of the following series. (Don't forget to check the endpoints).

   (a) $\displaystyle\sum_{j=1}^{\infty} \left(\frac{x}{2}\right)^j$

   (b) $\displaystyle\sum_{n=1}^{\infty} n^n (x-3)^n$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{\sqrt{k}}$

   (d) $\displaystyle\sum_{m=1}^{\infty} \frac{x^m}{m^2+1}$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-5)^n}{n \ln n}$

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k2^k}$

   (g) $\displaystyle\sum_{j=1}^{\infty} \frac{(3x)^j}{j}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{nx^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

2. Find the domain of the following functions.

   (a) $\displaystyle f(x) = \sum_{k=0}^{\infty} \frac{kx^k}{2k+1}$

   (b) $\displaystyle g(x) = \sum_{k=0}^{\infty} \frac{k^2(x+1)^k}{3^k}$

   (c) $\displaystyle h(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

   (d) $\displaystyle f(x) = \sum_{n=1}^{\infty} \frac{n(2x)^n}{\ln(n+1)}$

   (e) $\displaystyle g(x) = \sum_{m=0}^{\infty} \frac{m}{\sqrt{m+1}} (x-e)^m$

3. Find all values of x for which the series converges.

   (a) $\displaystyle\sum_{j=1}^{\infty} \frac{(-1)^j (x-5)^j}{j(j+1)}$

   (b) $\displaystyle\sum_{m=0}^{\infty} (-1)^m \frac{(x-1)^{4m}}{m!}$

   (c) $\displaystyle\sum_{m=0}^{\infty} \frac{3^m (x-2)^m}{m!}$

   (d) $\displaystyle\sum_{j=1}^{\infty} \frac{1}{j(x-4)^j}$

Concepts

1. The function $J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$ is called the *Bessel function of order 1.* Find its domain.
2. Let $R > 0$ be any constant.
   (a) Show that $\sum_{k=0}^{\infty} \frac{x^k}{R^k}$ converges on $(-R, R)$.
   (b) Show that $\sum_{k=1}^{\infty} \frac{x^k}{kR^k}$ converges on $[-R, R)$.
   (c) Show that $\sum_{k=1}^{\infty} \frac{x^k}{k^2R^k}$ converges on $[-R, R]$.
   (d) Concoct a power series that converges on $(-R, R]$.
3. For each of the given intervals, give an example of a power series that has the given interal as its interval of convergence.
   (a) $[-4, 4)$
   (b) $[-1, 5]$
   (c) $(-4, 0)$
4. Suppose the power series $\sum_{n=0}^{\infty} a_n(x-b)^n$ converges on $-11 \leq x < 17$.
   (a) What is the radius of convergence of the series?
   (b) Determine the value of b.
5. If $\sum_{m=0}^{\infty} a_m x^m$ converges for $x = 3$, what, if anything, can be said about the convergence at $x = 2$? Can anything be said about the convergence at $x = 5$?
6. If $\sum_{j=0}^{\infty} a_j(x-2)^j$ converges for $x = 6$, at what other numbers x must the series necessarily converge?
7. Use the techniques discussed in this section to find the interval of convergence of the series $\sum_{k=1}^{\infty} k^x$.

Maple

1. The power series $\sum_{k=1}^{\infty} \frac{x^k}{k}$ has radius of convergence 1. Plot the partial sum polynomials of degree 1, 2, 4, 6, 8, and 10 over the interval $[-2, 2]$. Is the interval of convergence of the series apparent? Explain. [Hint: experiment with different ranges (of y).]