MA114 Worksheet for Section 11.6

Skills

1. Use the power series for $\dfrac{1}{1-x}$ to produce a power series representation of each of the following functions.

   (a) $\dfrac{x^2}{1+x}$

   (b) $\dfrac{1}{1-x^2}$

   (c) $\dfrac{1}{(1+x)^2}$

2. For each of the following functions, use known power series to find a power series representation of the function and the radius of convergence of this power series. If you have time, use Maple to plot the function and the fifth-order polynomial that is a partial sum of the power series on the same axes.

   (a) $f(x) = \arctan(2x)$

   (b) $f(x) = 2^x = e^{x \ln 2}$

   (c) $f(x) = \ln(1+x^2)$

   (d) $f(x) = x^2 \sin x$

   (e) $f(x) = \dfrac{1}{2+x}$

   (f) $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ Hint: integrate a series you have alredy seen.

   (g) $f(x) = \sin(\sqrt{x})$

   (h) $f(x) = \cos^2 x = \frac{1}{2}(1+\cos(2x))$

3. Find the first four non-zero terms in the power series representation of each of the following functions.

   (a) $\dfrac{e^x}{1+x}$

   (b) $\ln(1+\sin x)$

4. Use power series to evaluate each of the following limits. Compare your results with those obtained using L'Hôpital's rule.

   (a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$

   (b) $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

   (c) $\displaystyle\lim_{x\to 0} \frac{e^x - 1}{x}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\ln(1+x) - x}{x^2}$

Concepts

1. Let $f(x) = \dfrac{1}{1+x^4}$.

   (a) Find a power series representation for f.

   (b) What is the interval of convergence of this series?

   (c) Use the series found above to evaluate $\displaystyle\int_0^{0.5} f(x)\,dx$ with an error no greater than 0.001.

2. (a) Find a power series representatnion of an antiderivative of e^{-x^2}.

(b) Use the result from above to estimate $\int_0^1 e^{-x^2}\,dx$ to within .005 of its exact value.

3. Use a power series to show that $x - \frac{1}{2}x^2 < \ln(1+x) < x$ for all $x \in (0,1)$.

4. Use a known power series to verfiy the following sums.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)} = \frac{1}{4}$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)(K=3)} = \frac{1}{18}$

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{(k+1)!} = 1$