MA114 Worksheet for Section 12.2

Skills

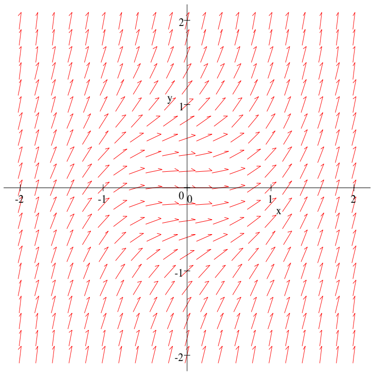
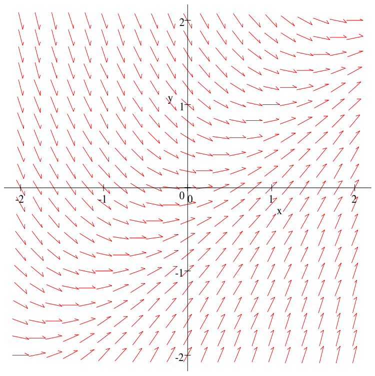
1. Match the slope fields below with the following differential equations. [Hint: consider where $y' = 0$.]

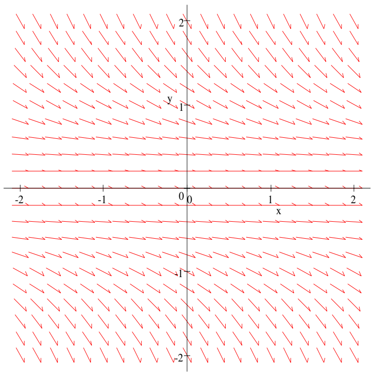
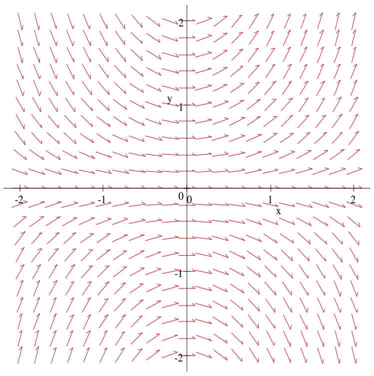
   (a) $y' = x^2 + y^2$

   (b) $y' = xy$

   (c) $y' = -\dfrac{y}{2}$

   (d) $y' = x - y$

Concepts

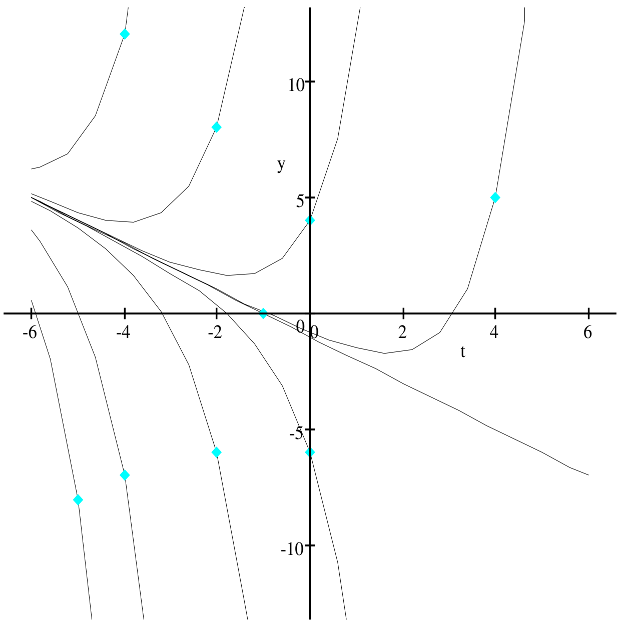
1. This problem concerns the differential equation $y' = y + t$.

   (a) Verify by substitution that $y = Ce^t - t - 1$ is a solution for the given differential equation.

   (b) Find the value of C that solves the IVP $y' = y + t; y(0) = 0$.

   (c) Below are graphs of several functions of the form $y = Ce^t - t - 1$ (for different values of C). One of the curves is a straight line. What is the equation of this line? What value of C does it correspond to? Verify that this line is a solution of the differential equation.

   (d) On the above axis draw the line $y + t = 0$ (use a sharp pencil to avoid drawing fat lines). This line crosses several solution curves at special points of interest. What is so special about these points?



(e) All the solutions seem to "converge" in the leftward direction toward the line $y = -t - 1$. To explain this phenomenon, show that if $y(t)$ is any solution to the differential equation that

$$\lim_{t \to -\infty} (y(t) - (-t - 1)) = 0.$$

Maple

1. Consider the differential equation $y' = \dfrac{x - y}{x + y}$.

   (a) Use *dfieldplot* to draw the slope field for the equation.

   (b) Use the slope field to draw a rough sketch of the solution to this differential equation that satisfies the initial conditon $y(0) = 1$. Then use the *phaseportrait* command to check your sketch.

   (c) Use *dsolve* to find the exact solution of the initial value problem.

2. Use Maple to plot the slope field for each of the given differential equations. Then have Maple plot 5 typical solutions for each differential equation.

   (a) $y' = \dfrac{x}{y}$

   (b) $y' = xe^{-y}$

   (c) $y' = \sin y$