MA114 Worksheet for Section 12.3

Skills

1. Use Euler's method to find estimates $y_1, y_2, \ldots, y_{10}$ of the values of the actual solution at $x_1, x_2, \ldots x_{10}$ over the given interval with step size h. Make a table similar to the one at the end of example 1 in the book. (This should be done using your calculator).

   (a) $y' = x + y, y(0) = 0; I = [0, 1]; h = 0.1$

   (b) $y' = xy, y(0) = 1; I = [0, 2]; h = 0.2$

   (c) $y' = y + 1, y(0) = -1; I = [0, 2]; h = 0.2$

Concepts

1. Write a differential equation that is a mathematical model of the situation described. Then use Euler's method to approximate a solution through the given initial condition. Use $h = .1$ and approximate 10 equally spaced points starting at the initial point.

   (a) The rate of change (with respect to time) of a population P is proportional to the square root of the current population. The initial population at time $t = 0$ years is 1000.

   (b) The rate of change of the velocity of a coasting motorboat is proportional to the square of the velocity. A time $t = 3$ seconds the boat is travelling at 30 mph.

Maple

1. For each IVP, use Euler's method with step sizes $h = 0.1, 0.02, 0.004$, and 0.0008 to approximate the values of the solution at 10 equally spaced points starting at the initial point.

   (a) $y' = x^2 + y^2, y(0) = 0$

   (b) $y' = \ln y, y(1) = 1$

   (c) $y' = \dfrac{x}{1 + y^2}, y(-1) = 1$

2. Consider the initial value problem $y' = 1 - x + 4y, y(0) = 1$.

   (a) Using Euler's method with a step size of $h = 0.05$, approximate $y(0.2)$.

   (b) Use Maple to find the exact solution of the above initial value problem. Evaluate this solution at $x = .02$ and compare your answer to your numerical result.