MA114 Worksheet for Section 12.4

Skills

1. Find general solutions of the following differential equations.

   (a) $\dfrac{dy}{dx} = xy^3$

   (b) $y\dfrac{dy}{dx} = x(y^2+1)$.

   (c) $y^3\dfrac{dy}{dx} = (y^4+1)\cos x$.

   (d) $\dfrac{dy}{dx} = \dfrac{1+\sqrt{x}}{1+\sqrt{y}}$.

   (e) $x\dfrac{dy}{dx}\tan y = x$.

   (f) $\dfrac{dy}{dx} = 2x\sqrt{y-1}$.

   (g) $\dfrac{dy}{dx} = xy + x - 2y - 2$.

2. Find solutions of the following initial value problems.

   (a) $\dfrac{dy}{dx} = ye^x; y(0) = 2e$.

   (b) $\dfrac{dy}{dx} = 3x^2(y^2+1); y(0) = 1$.

   (c) $2y\dfrac{dy}{dx} = x(x^2-16)^{-\frac{1}{2}}; y(5) = 2$.

   (d) $\dfrac{dy}{dx} = 4x^3y - y; y(1) = -3$.

   (e) $\dfrac{dy}{dx}\tan x = y; y\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$.

   (f) $x\dfrac{dy}{dx} - y = 2x^2y; y(1) = 1$.

   (g) $\dfrac{dy}{dx} + 1 = 2y; y(1) = 1$.

Concepts

1. A thermometer reading 4°C is brought into a room where the temperature is 30°C. If the thermometer reads 10°C after 2 minutes, determine the temperature reading 5 minutes after the thermometer is first brought into the room.

2. Newton's Law of Cooling also applies to heating. A 5 pound roast, initially at 50°F, is put into a 375°F oven at 5:00. The temperature of the roast after 75 minutes is 125°F. When will the roast be 150°F (medium rare)?

3. A certain piece of dubious information about phenylthiourea in the drinking water began to spread one day in a city of population 100,000. After one week, 10,000 people had heard this rumor. How long will it be before half the population of the city have heard this piece of information?

4. Atmospheric pressure is a function of altitude above sea level and is given by the equation $\dfrac{dP}{da} = \beta P$, where β is a constant. The pressure is measured in millibars (mb). At sea level, $(a = 0)$, $P(0) = 1013.25$ mb, which means that the atmosphere at sea level will support a column of mercury 1013.25 millimeters high at a standard temperature of 15°C. At an altitude of $a = 1500$ meters, the pressure is 845.6 millibars.

(a) In California, the highest and lowest points are Mount Whitney (4418 meters) and Death Valley (86 meters below sea level). What is the difference in their atmospheric pressures?

(b) What is the atmospheric pressure at Mount Everest (8848 meters)?

(c) At what elevation is the atmospheric pressure equal to 1 millibar?

5. Let $y(t)$ and $V(t)$ be the height and volume of water in a tank at time t. If water leaks through a hole with area a at the bottom of the tank, then *Torricelli's Law* says that

$$\frac{dV}{dt} = -a\sqrt{gy}$$

where g is the acceleration due to gravity.

(a) Suppose the tank is cylindrical with height 6 feet and radius 2 feet, and the hole is circular with radius 1 inch. If we take $g = 32$ feet per second squared, find the differential equation that defines $y(t)$.

(b) Solve this equation to find the height of the water at time t, assuming the tank is full at time $t = 0$.

(c) How long will it take the tank to drain completely?

6. Early one morning it began to snow at a constant rate. At 7:00 in the morning a snow plow set off to clear a road. By 8:00 it had travelled 4 miles, and by 9:00 it had travelled an additional 3 miles. Assuming that the snowplow clears snow from the road at a constant rate (in cubic feet per hour), when did it start snowing? (Hint: The answer is 4:27).