MA114 Worksheet for Section 5.1

Skills

1. Given $\int_0^1 f(x)dx = 3$, $\int_1^2 f(x)dx = 2$, and $\int_0^3 f(x)dx = -1$, find the values of:

   (a) $\int_0^2 f(x)dx$

   (b) $\int_1^3 f(x)dx$

   (c) $\int_2^3 f(x)dx$

2. Suppose that $\int_0^2 f(x)dx = 1$, $\int_0^2 g(x)dx = 5$, and $\int_0^2 h(x)dx = -\frac{5}{2}$. Evaluate as many of the following as you can by using the properties of integrals. Note which integrals cannot be evaluated by the properties.

   (a) $\int_0^2 (f(x) + g(x))dx$

   (b) $\int_0^2 f(x)g(x)dx$

(c) $\int_0^2 2g(x)dx$

(d) $\int_0^2 (g(x) - h(x))dx$

(e) $\int_0^2 \frac{g(x)}{h(x)}dx$

3. Show that $0 \leq \int_0^\pi x \sin x dx \leq \frac{\pi^2}{2}$.

4. Evaluate $\int_0^1 \sqrt{1-x^2}dx$

Concepts

1. Let f be a continuous function on the interval [-5,10] and let $g(x) = f(x) + 2$. If $\int_{-5}^{10} f(t)dt = 4$, what is $\int_{-5}^{10} g(u)du$?

2. Suppose that f is a continuous function. If the average value of f over the interval $[-3, 1]$ is 2 and the average value of f over the interval $[-3, 7]$ is 5, what is the average value of f over the interval $[1, 7]$?

3. Suppose that f is a function such that $\displaystyle\int_0^3 f(x)dx = -1$.

   (a) Suppose that f is an *even* function. Explain why $\displaystyle\int_{-3}^3 f(x)dx = -2$.

   (b) If f is an *odd* function, what is the value of $\displaystyle\int_{-3}^3 f(x)dx$? Explain.