# MA114 Worksheet for Section 5.3

**Skills**

1. State the Fundamental Theorem of Calculus (either the "Formal Version" or the "Second Version." )

2. Find an antiderivative $F(x)$ with $F'(x) = f(x)$ and the given intitial conditions.

   (a) $f(x) = x^2$; $F(1) = \frac{3}{5}$.

   (b) $f(x) = \sqrt{x}$; $F(4) = 1$.

   (c) $f(x) = \sin(\frac{x}{2})$; $F(\frac{\pi}{3}) = -1$.

   (d) $f(x) = \dfrac{1}{x}$; $F(1) = 1$.

(e) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$; $F(\frac{\sqrt{2}}{2}) = e$.

3. Let $F(x) = \displaystyle\int_0^x f(t)dt, G(x) = \int_1^x f(t)dt$, and $H(x) = \displaystyle\int_{-2}^x f(t)dt$.

   (a) Suppose that $f(x) = 4$. Find symbolic expressions for the functions $F, G$, and $H$.

   (b) Suppose that $f(x) = 7x$. Find symbolic expressions for the functions $F, G$, and $H$.

   (c) Suppose that $f(x) = 7x + 4$. Find symbolic expressions for the functions $F, G$, and $H$.

4. Evaluate each of the following definite integrals using the fundamental theorem of Calculus.

   (a) $\displaystyle\int_1^4 \left(x^2 + x^{5/2}\right) dx$

   (b) $\displaystyle\int_0^\pi \sin x dx$

(c) $\displaystyle\int_{-1}^{4} \frac{dx}{2+x}$

(d) $\displaystyle\int_{-b}^{b} x^3 dx$

**Concepts**

1. Recall the definition of the average value of a function from section 5.1.

   (a) Find the average values of $f(x) = x$, $f(x) = x^2$, and $f(x) = x^3$ over the interval [0,1].

   (b) From the pattern that is established in part (a), what is the average value for $f(x) = x^n$, for an integer $n \geq 1$?.

   (c) What does the answer to part (b) imply about the average value of $f(x) = x^n$, as $n$ gets larger and larger? Can you explain this from the graph of $f(x) = x^n$?

2. What is wrong with the following calculation?

$$\int_{-1}^{3} \frac{1}{x^2}\, dx = \left.\frac{x^{-1}}{-1}\right]_{-1}^{3} = -\frac{1}{3} - 1 = -\frac{4}{3}$$

3. A company is planning to phase out a product because demand for it is declining. Demand for the product is currently 800 units/month and dropping by 10 units/month each month. To maintain customer and employee relations, the company has announced that it will continue to produce the product for one more year. At the present time, the company is producing the product at a rate of 900 units/month, and 1680 units of the product are in inventory.

   (a) Give expressions for the demand and production rates as functions of time (take $t = 0$ as the present).

   (b) Let $D(t)$ be the demand rate for the product. Explain why $\int_0^t D(s)ds$ is the total demand for the product in $t$ months' time.

   (c) Give an expression for the inventory at time $t$.

   (d) The company would like to reduce production at a constant rate of $R$ units per month, with $R$ chosen so as to reduce its inventory to zero at the end of 12 months. What should $R$ be to reduce the inventory to zero at $t = 12$?

4. Suppose that $F$ is an antiderivative of a differentiable function $f$.

   (a) If $F$ is increasing on $[a, b]$, what is true about $f$?

(b) If $f$ is negative on $[a, b]$, what is true about $F$?

(c) If $f'$ is positive on $[a, b]$, what is true about $F$?

(d) If $F$ is concave down on $[a, b]$, what is true about $f'$?

(e) Suppose that $G$ is another antiderivative of $f$. What relationship exists between $F$ and $G$?