MA114 Worksheet for Section 5.4

Skills

1. Although the Fundamental Theorem is a powerful tool, it cannot always be applied since some functions do not have antiderivatives which we can compute. In these cases, the best we can do is estimate the value of the integral by using approximations. Using your calculator, sketch each of the following curves. Estimate each of the following integrals using the left hand rule, the right hand rule, the mid-point rule, and the trapezoid rule, for $N = 4$ equal subdivisions.

   (a) $\displaystyle\int_0^1 \frac{1}{1+x^4}dx$

   (b) $\displaystyle\int_0^1 \ln(1+\sqrt{x})dx$

   (c) $\displaystyle\int_0^{\sqrt{\pi}} \cos(x^2)dx$

2. For each of the integrals in the question above, use limits and sigma notation to write an expression for the exact indicated area. Do not try to evaluate the expression!

3. Express the following sums in sigma notation:

(a) $2 + 5 + 10 + 17 + \cdots + 101$

(b) $-1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} + \cdots + \frac{1}{35}$

(c) $\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \cdots + \frac{11}{2^{12}}$ (Hint: Some of the fractions have been reduced).

4. Express the following sums,written in sigma notation, in the form above:

(a) $\sum_{i=1}^{10} \frac{i-1}{i^2}$

(b) $\sum_{i=1}^{9} \frac{(-1)^i}{2i}$

(c) $\sum_{i=0}^{8} \frac{2^i}{i!}$ where $0! = 1$ by definition.

Concepts

1. Let $f(x) = x^2 - 1$. Consider the region bounded by the graph of f, the x-axis, and the line $x = 2$. Divide the interval [0,2] into 8 equal subintervals. Draw a picture to help you answer the following.

   (a) Obtain a lower estimate for the area of the region by using the left endpoint of each subinterval.

   (b) Obtain an upper estimate for the area by using the right enpoint of each subinterval.

   (c) Find an approximation for the area that is better than either of the answers obtained in parts (a) and (b).

   (d) Determine whether the answer in part (c) is larger or smaller than the true area, and supply a reason.

2. This problem concerns the region under the graph $f(x) = \sqrt{16 - x^2} + 1$ from $x = 0$ to $x = 4$. You will be asked to find Riemann sums for the area of this region by picking sample points in various ways.

   (a) Sketch the graph of f from $x = 0$ to $x = 4$.

(b) Divide the inteval [0,4] into four subintervals each 1 unit wide. Pick x in the middle of each subinterval, find the corresponding values of $f(x)$, and then calculate the Riemann sum for these choices of x.

(c) Find a different Riemann sum by picking the value of x in each subinterval where $f(x)$ is the highest.

(d) Repeat part c, using the lowest value of $f(x)$ in each of the subintervals.

(e) Find an approximation for the area by averaging the two Riemann sums in parts (c) and (d). Is this value the same as what you calculated in part (b)? Why or why not? Compare this value to the average of the values computed in (c) and (d). Is it the average of those values. Why or why not?

(f) A closer approximation to the area can be found by dividing the region into narrower strips. Divide the interval [0,4] into eight subintervals, each 1/2 unit wide. Using the midpoint of each subinterval, find the corresponding values of $f(x)$, and use them to calculate a new Riemann sum.

(g) Using the concept of limit, how might you define the exact area of this region?

(h) Calculate the exact area using familiar area formulas. How do the approximations you have calculated compare with the exact area?

3. Consider the following table of values of a continuous function f at different values of x:

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	.13	.18	.26	.36	.25	.14	.02	$-.11$	$-.28$	$-.33$

(a) From the given data, find *two* estimates of $\int_1^{10} f(x)dx$.

(b) Obtain a different estimate for the integral by taking an average value of f over each subinterval.

4. Evaluate the following infinite limits by expressing each of them as a definite integral and then evaluating the integral.

(a) $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^3 + \left(\frac{2}{n}\right)^3 + \cdots + \left(\frac{2n}{n}\right)^3\right]$

(b) $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{n+1}{n}\right) + \left(\frac{n+2}{n}\right) + \cdots + \left(\frac{2n}{n}\right)\right]$

(c) $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n+1}\right)^3 + \left(\frac{2}{n+2}\right)^3 + \cdots + \left(\frac{n}{2n}\right)^3\right]$