MA114 Worksheet for Section 6.2

Skills

1. Evaluate the following integrals:

   (a) $\int (x^2-3)^5 x\,dx$

   (b) $\int (x^2+2x+5)^3 x\,dx$

   (c) $\int (x^4-3x^2+7)^{11}(2x^3-3x)\,dx$

   (d) $\int (y-3)^7\,dy$

   (e) $\int \dfrac{dy}{4-y}$

   (f) $\int \dfrac{dy}{\sqrt{y+3}}$

   (g) $\int \cos(z)(1-\sin(z))^9\,dz$

   (h) $\int e^{3z+1}\,dz$

   (i) $\int \dfrac{t-1}{t^2+1}\,dt$

   (j) $\int t\sqrt{1-t^2}\,dt$

   (k) $\int \tan(3t)\,dt$

   (l) $\int \sin^5\theta\,d\theta$

   (m) $\int \dfrac{d\theta}{(\theta-4)^3}$

   (n) $\int \theta\sin(\theta^2)\,d\theta$

Concepts

1. Al and Betty are having an argument as to the value of $\int \sec^2(x)\tan(x)dx$. Al makes the substitution $u=\sec(x)$ and one answer and Betty makes the substitution $u=\tan(x)$ and gets a different answer. Find the answers that Al and Betty found and explain what is going on so that they will stop arguing.

2. If we assume that the wind resistance is proportional to the square of velocity, then the downward velocity, v, of a falling body is given by,

$$v=\sqrt{\frac{g}{k}}\left(\frac{e^{t\sqrt{gk}}-e^{-t\sqrt{gk}}}{e^{t\sqrt{gk}}+e^{-t\sqrt{gk}}}\right)$$

   Use the substitution $w=e^{t\sqrt{gk}}+e^{-t\sqrt{gk}}$ to find the height, h, of the body above the surface of the earth as a function of time. Assume that g and k are constants and that the body starts at a height h_0.

3. Evaluate the following integrals (these could easily be skills problems as well):

   (a) $\int \dfrac{\ln x}{x}\,dx$

(b) $\displaystyle\int \frac{dx}{x \ln x}$

(c) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

(d) $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

(e) $\displaystyle\int \frac{1}{u^2}\sqrt{1+\frac{1}{u}}\,du$

(f) $\displaystyle\int \tan^2 u \sec^2 u\,du$

(g) $\displaystyle\int \sqrt[3]{u^3+1}u^5\,du$

(h) $\displaystyle\int \frac{u^2 \sin u}{1+u^6}\,du$

(i) $\displaystyle\int \cos\mu \ln(\sin\mu)\,d\mu$

(j) $\displaystyle\int \frac{cos\sqrt{\mu}}{\sqrt{\mu}}\,d\mu$

(k) $\displaystyle\int \sec\mu \tan\mu \sqrt{1+\sec\mu}\,d\mu$

4. Use substitution to verify the following identities:

(a) $\displaystyle\int_b^a f(-x)\,dx = \int_{-a}^{-b} f(x)\,dx$

(b) $\displaystyle\int_b^a f(x+c)\,dx = \int_{b+c}^{a+c} f(x)\,dx$

(c) $\displaystyle\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$

5. (a) Use the substitution $u = \pi - x$ to show that

$$\int_0^\pi x f(\sin x)\,dx = \frac{\pi}{2}\int_0^\pi f(\sin x)\,dx$$

(b) Use this result to evaluate

$$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx$$