MA114 Worksheet for Section 7.1

Skills

1. Let $f(x) = \sqrt{\ln x}$.
   (a) Sketch the graph of $f(x)$ over the interval $(0, 10)$. You may use your calculator or Maple to help you make your sketch.
   (b) Can you apply the error bound formulas to $\int_0^5 f(x)\,dx$? Why or why not?
   (c) If the error bound theorem can be applied, use it to find the smallest n so that $|\int_0^5 f(x)\,dx - R_n| \le .001$.
2. Let $f(x) = x^2 + 1$ and $I = \int_0^1 f(x)\,dx$.
   (a) Sketch the graph of $f(x)$ over the interval $[0, 1]$. Include the rectangles used in estimating I with L_4.
   (b) Will L_4 overestimate or underestimate I?
   (c) Compute L_4 *by hand.* (You may use a calculator, but not Maple).
   (d) Compute I using the fundamental theorem.
   (e) Compute $|I - L_4|$. Compare this to the estimate of $|I - L_4|$.
   (f) Compute the smallest n so that $|I - L_n| \le .000005$.
   (g) Compute the smallest m so that $|I - T_m| \le .000005$.
   (h) Will it require less work to compute T_m or L_n?
3. Let $I = \int_0^2 f(x)\,dx$. Suppose we know that $f(0) = 1, f(2) = 2$ and $L_{20} = 1.1134$.
   (a) What is R_{20}?
   (b) What is T_{20}?
   (c) Does this make you feel any differently about your answer to the last part of the previous question? Explain.

Concepts

1. Let $I = \int_a^b f(x)\,dx$ and suppose f is increasing on $[a, b]$.
   (a) Explain why $L_n \le M_n \le R_n$.
   (b) Rank L_n, R_n, and T_n in a similar fashion.
2. Let $I = \int_3^8 f(x)\,dx$. Indicate which of the following statements must be true, cannot be true, or might be true but cannot be determined.
   (a) If $L_{10} < I$ and $f'(x) > 0$ for all $x \in [3, 8]$ then $L_{20} < I$.
   (b) If f is monotone on $[3, 8]$, $f(3) = 5$, and $f(8) = 1$, then $|I - T_{1000}| \le .05$.
   (c) If f is monotone on $[3, 8]$, $f(3) = 5$, and $f(8) = 1$, then $|I - L_n| > 0.1$ unless $n > 200$.
   (d) If $|R_{10} - I| < .05$, then $|R_{20} - I| < .05$.
3. Let $f(x) = \cos(x^2)$.
   (a) Plot or sketch the graph of $f(x)$ over $[0, 3]$.

(b) Suppose we use L_n to estimate $I = \int_0^3 f(x)\,dx$. Can we use the error bound theorem to find a bound for $|L_n - I|$?

(c) Can we use the error bound theorem to bound $|L_n - \int_0^1 f(x)\,dx|$?

(d) What is the largest a so that we may use the theorem to bound $|L_n - \int_0^a f(x)\,dx|$?

(e) Assuming you've found a, can you find the largest b so that we may use the theorem to bound $|L_n - \int_a^b f(x)\,dx|$?

(f) Propose a method of estimating $I = \int_0^3 f(x)\,dx$ with an error no greater than ± 0.05 by using L_n on subintervals of $[0, 3]$.

4. If f is a positive function and $f''(x) < 0$ for all $a \le x \le b$ show that $T_n \le \int_a^b f(x)\,dx$. What happens when f is a negative function?

5. Use R_n and the following data to estimate $\int_1^{3.2} f(x)\,dx$.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2
$f(x)$	4.9	5.4	5.8	6.2	6.2	7.0	7.3	7.5	8.0	8.2	8.3	8.3

6. The speedometer reading on a car was observed at 1 minute intervals and recorded in the following chart. Use T_n to estimate the distance travelled by the car.

t	0	1	2	3	4	5	6	7	8	9	10
speed	40	42	45	49	52	54	56	57	57	55	56

Maple

1. Our goal is to find the area of the region under the graph of $y = x \sin x$ from $x = 0$ to $x = \pi$.

   (a) Plot the *leftbox* approximation of the region for $n = 10$ (ten rectangles). Then find the decimal value of the corresponding *leftsum* approximation.

   (b) Repeat the leftsum calculation for $n = 20, 40$, and 80. The results obtained should suggest a conjecture as to the exact value of the area.

   (c) Use Maple to find a "closed form" expression for $LEFT(n)$, then invoke the *limit* command to find the exact value of the area of the region. Be patient, this may take several minutes. Note that certain versions of Maple on certain machines will choke on the *limit* command.