MA114 Worksheet for Section 7.2

Skills

1. For each of the following integrals compute left and right approximations using $n = 6$ equal subintervals.
   (a) $\int_1^4 \sqrt{x}\, dx$
   (b) $\int_0^\pi x \sin x\, dx$
   (c) $\int_0^1 \frac{dx}{1+x^2}$
2. For each of the following integrals find a general expression for L_n and R_n for any n.
   (a) $\int_1^3 2\, dx$
   (b) $\int_1^4 x\, dx$
   (c) $\int_0^2 x^2\, dx$
3. For each of the following integrals, find the smallest value of n for which Theorem 2 guarantees that R_n approximates the value within ± 0.001.
   (a) $\int_0^1 \frac{1}{1+x^3}\, dx$
   (b) $\int_0^{\frac{\pi^2}{2}} \sin \sqrt{x}\, dx$

Concepts

1. Consider $\int_1^4 \frac{e^x}{x}\, dx$. How does the bound on the approximation error made by L_n given by Theorem 2 compare with the bound given by Theorem 1?
2. Let $I = \int_0^2 \sqrt{4-x^2}\, dx$.
   (a) Explain why $I = \pi$. Hint: Think geometrically.
   (b) Compute L_{10} and $|\pi - L_{10}|$.
   (c) Explain why Theorem 2 does not apply here.
   (d) Compare the error with the bound given by Theorem 1.
3. Suppose $0 < f'(x) < g'(x)$ for every $x \in [a, b]$. Is it true that $|L_n - \int_a^b f(x)\, dx| < |L_n - \int_a^b g(x)\, dx|$?

Maple

1. Let $I = \int_0^1 \frac{4}{1+x^2}\, dx$.
   (a) Verify (you should be able to do this by hand) that $I = \pi$.
   (b) Compute error bounds on $|L_n - I|$ using both Theorem 1 and Theorem 2. For Theorem 2, note that the derivative of $\frac{4}{1+x^2}$ is a continuous function on a closed interval and must attain a maximum value somewhere on that interval. You should know how to find it!
   (c) Use Maple to compute L_n for various values of n, say 20, 50, 100, and 200, and thus find approximations for π.
   (d) See how close an approximation you can get to π by trying larger and larger values of n. Notice that larger values of n take longer for Maple to compute, so don't just try a huge value of n right away.