MA114 Worksheet for Section 7.3

Skills

1. For each of the following integrals, compute L_{10}, M_{10}, and T_{10}. Compare these estimates with the exact values and check that the bound on the magnitude of the approximation error given in Therorem 3 is satisfied.

   (a) $\displaystyle\int_1^4 x\,dx$

   (b) $\displaystyle\int_0^3 x^2\,dx$

   (c) $\displaystyle\int_0^\pi \sin x\,dx$

   (d) $\displaystyle\int_2^1 \frac{1}{x}\,dx$

2. For each of the following integrals, find the smallest value of n which Theorem 3 guarantees an error of less than ± 0.0001.

   (a) $\displaystyle\int_0^1 x^3\,dx$

   (b) $\displaystyle\int_0^2 e^x\,dx$

3. How large do we have to choose n so that both approximations T_n and M_n to the integral $\displaystyle\int_0^1 e^{-x^2}\,dx$ are accurate to within ± 0.00001?

Concepts

1. Use the trapezoid rule and the midpoint rule to approximate the given integrals with the given value of n. (Round your answer to six decimal places).

   (a) $\displaystyle\int_{-1}^1 \sqrt{1+x^3}\,dx; n = 8.$

   (b) $\displaystyle\int_0^1 \cos(x^2)\,dx; n = 4$

   (c) $\displaystyle\int_{\pi/2}^\pi \frac{\sin x}{x}\,dx; n = 6$

2. Find the approximations T_n and M_n for the following integrals and given values of n. Then compute the correspoding errors. What observations can you make? In particular, what happens to the errors when n is doubled.

   (a) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx; n = 4, 8$

   (b) $\displaystyle\int_1^4 \sqrt{x}\,dx; n = 6, 12$

3. A log 10 meters long is cut at 1 meter intervals and its cross sectional areas A (at a distance x meters from the end of the log) are listed in the following table. Use the trapezoid rule to estimate the volume of the log.

x	0	1	2	3	4	5	6	7	8	9	10
$A(x)$	0.68	0.65	0.64	0.61	0.58	0.59	0.53	0.55	0.52	0.50	0.48

Maple

1. This problem is designed to illustrate some of the general issues and techniques involved in finding good numerical approximations of the definite integral.

   (a) Sketch (plot) the graph of $f(x) = \dfrac{1}{\sqrt{1+x^4}}$ and examine the concavity on $[-1, 1]$.

   (b) Find values (decimal) of M_{10} and T_{10} which approximate $\displaystyle\int_{-1}^{1} f(x)\,dx$. What does the concavity condition alluded to above tell you about the exact value of the integral in relation to the two approximations?

   (c) Compute M_n and T_n for larger and larger values of n until you are satisfied that you have computed an approximation of the integral exact to 6 decimal places.