MA114 Worksheet for Section 7.4

Skills

1. For each of the following integrals find S_2. Compare this value to the actual value of the integral. (See problem 1 under concepts).

   (a) $\int_1^3 x+1\,dx$

   (b) $\int_0^4 x^2-2x\,dx$

   (c) $\int_{-1}^1 x^3+1\,dx$

2. Let $I=\int_0^2 e^{x^2}\,dx$.

   (a) Compute S_4 using only a scientific calculator.

   (b) Bound the approximation error made by S_4.

   (c) What is the smallest n necessary to guarantee that S_n approximates I to within 10^{-5}?

3. Let $I=\int_0^1 \cos(100x)\,dx$.

   (a) Compute the value of I exactly using the fundamental theorem.

   (b) Compute S_{10} and find the approximation error.

   (c) Find an upper bound on the approximation error using Theorem 3. Why is this bound so enormous?

Concepts

1. Show that if f is a polynomial of degree three or lower (i.e, let $f(x)=ax^3+bx^2+cx+d$), the Simpson's rule gives the exact value of $\int_{x_0}^{x_1} f(x)\,dx$.

2. Suppose S_{10} is the first Simpson sum guaranteed to approximate a certain definite integral I within 10^{-2}.

   (a) How big is the approximation error for S_{100}?

   (b) What is the smallest value of n such that $|S_n-I|<10^{-10}$?

3. Show that $\frac{2}{3}M_n+\frac{1}{3}T_n=S_{2n}$.

4. A valve is opened on a tank of pressurized gas. As gas passes through the valve, the pressure in the tank decreases, thus causing less gas to be forced through the valve. The rate at which the gas passes through the valve is $r(t)=\dfrac{4}{1+\sqrt{t+1}}$ cubic centimeters per second. Using Simpson's rule, estimate the amount (volume) of gas that has been released from the tank after 30 seconds.

Maple

1. The function $G(x)=\dfrac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$ has many mathematical applications. It is commonly refered to as the "Gaussian" in honor of Karl Friedrich Gauss (1777-1855). Since the antiderivative of G cannot be expressed in terms of elementray functions, the area over any finite interval must be approximated by numerical techniques. Tables of values appear in many textbooks and handbooks of mathematical tables and formulas.

   (a) Use Maple to plot this curve.

(b) Use Simpson's rule with $n = 60$ to obtain an approximation of the area under the curve over the interval $[-3, 3]$. Similarly obtain an approximation over $[-5, 5]$ with $n = 100$, and $[-10, 10]$ with $n = 200$.

(c) Make a guess as to the behavior of $\int_{-M}^{M} G(x)\,dx$ as $M \to \infty$. If you have time, try to support your guess by approximating the integral for large values of M.

(d) Let $G_k(x) = kG(kx)$ for $k = 1, 2, 3, \ldots$. Plot graphs of G_{10}, G_{20} and G_{30} all on the same axes. Then use Simpson's rule with $n = 100$ to obtain approximations of the area under the graph of each of these functions over the interval $[-0.2, 0.2]$.

(e) Can you guess what is happening to $\int_{-0.2}^{0.2} G_k(x)\,dx$ as $k \to \infty$? If you have time, try to support your guess by approximating the integral for large values of k.

Note: As in previous assignments, don't jump to values of M or k that are too large too quickly. Experiment with values "close" to the ones you already know about and gradually work your way up to large ones. For example, you might wish to try $M = 20, 50$ and 100 instead of starting with $M = 250$.